The Mathematical Model Development of Modular Three-Link Trailer Truck Train Motion

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Abstract. In the case when a multi-link vehicle consists of 3 links or more, the study of their movement is significantly complicated, since the number of factors that influence the nature of the movement of the constituent links of the road train significantly increases. During the movement of a multi-link vehicle the force interaction between its adjacent links significantly affects the entire road train, which can lead to significant deviations of its links from the movement direction set by the tractor truck. Road trains are a type of transport with increased danger, therefore, when designing and researching multi-link vehicles, it is necessary to conduct detailed theoretical and numerous experimental studies of the dynamics of their movement, which will become the basis for predicting their behavior in real conditions. Proceeding from the above, the layout scheme of a road train with 3-axle tractor, 2-axle trolley and 3-axle semi-trailer as universal and promising (developed by the companies "Scania" and "Krone") was chosen to study its movement and develop a mathematical model of movement. This model will allow us to theoretically predict and determine the influence of our layout scheme and driving modes of 3-link road train on its operational properties. At first time the subject of mathematical modeling became the three-link road truck train with a two-axle trolley and a coupling point located outside the tractor base. This configuration was not the object of in-depth analysis in previous works. This study is comprehensive in nature and demonstrates a number of aspects that distinguish the proposed approach against the background of existing scientific developments in the field of transport system dynamics.

Keywords: Trailer · Semi-Trailer · Modular Three-Link Trailer Truck · Multi-Link Vehicle · Trailer Link · Rolling Cart · Operational Properties · Maneuverability

1 Introduction

Solving problems related to the safety of three-link road trains is not solved only by the use of electronic systems: automatic wheel lock elimination like anti-lock braking system (ABS), brake force distribution system (EBD), stability control system (ESP),

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and others. Such systems can significantly improve the braking properties of vehicles, increase stability and ensure the preservation of all parameters of maneuverability of a three-link road train in extreme driving modes.

Deterioration of the maneuverability of a road train can lead to fatal consequences. Therefore, there is a need to study the influence of layout and different operational factors on the maneuverability of a three-link road train.

Solving these problems is impossible without developing the mathematical model of the movement for modular three-link road train, which would take into account the main kinematic and geometric relationships, normal reactions of the supporting surface and lateral forces on the wheels of the links axles, with taking into account their redistribution along the axles and sides during curved and straight movement, as well as the angles of deflection the tractor wheels, semi-trailer and trolley.

The study analyzed the existing mathematical models used to study the road trains dynamics of movement. Most of them have a simplified structure, which limits their accuracy in cases of complex coupling schemes or operation in real road conditions. The works of previous researchers mainly considered layout schemes of 2-link truck trains consisting of tractor trucks with semi-trailer or three-link truck trains consisting of tractor trucks with a semi-trailer and a trailer. In our study, for the first time, the subject of mathematical modeling was a three-link truck train with a two-axle trolley and a coupling point located outside the tractor base. This configuration has not been the subject of in-depth analysis in previous works.

Taking into account the design features of the proposed scheme for 3-link road train makes the model practically applicable for optimizing the designs of similar road trains used in the real sector of the economy, in the agricultural industry or in transport. The analytical dependencies we obtained can be integrated into software for engineering analysis of motion dynamics.

The research aim of our scientific work is developing the mathematical model of a modular three-link trailer truck train movement consisting of a "tractor truck - two-axle trolley with fixed axles (dolly) - three-axle semi-trailer", which will be used to further study the influence of operational and layout factors on the maneuverability for specified trailer truck train and predict its behavior in difficult road conditions.

The object of the research is a modular three-link trailer truck train.

The subject of the research is external and internal factors which influence on the maneuverability, stability and other operational characteristics of a road train to predict its behavior in difficult road conditions.

The practical significance of the work lies in the fact that the developed mathematical model of a modular three-link trailer truck train movement will allow us to theoretically predict the influence of the driving modes on its operational properties.

2 Literature Review

The research of operational properties of road trains is devoted to the works of D. Antonov, S. Atayev, V. Verbytsky, Ya. Zakin, E. Ibragimov, A. Kolpakov, M. Kryshenia, L. Lobas, M. Michke, S. Margolis, V. Poliakov, V. Sakhno, Ya. Farobin, D. Ellis, Benjamin Esterl & Torsten Butz, E.F. Kurtz, Alejandro Bravo-Doddoli, Luis C. García-Naranjo, Ali Keymasi Khalaji & Seyed Amir Ali Moosavian, Xiang Kang та Wei Deng, Jonathan Darling, David Tilley & Bo Gao, Mattia Zanchetta, Davide Tavernini and many other national and foreign scientists. Analysis of publications shows that the regularities of movement of two-link road trains are quite well studied.

However, there are relatively few works devoted to the study of the operational properties of multi-link road trains (primarily three-link ones), and in this regard, many problems remain that have not yet been studied in sufficient depth. Thus, in work [1], the movement of 3-link road trains with different layout schemes in different driving modes was considered. The authors of work [2] conducted a comparative assessment of 3-link road trains with different layout systems from a maneuverability point of view. Work [3] is devoted to the study of the maneuverability of three-link metrobuses.

In works [4, 5] the influence of operational factors and design on maneuverability indicators of the metrobus was considered. The authors of work [6] conducted a research of the maneuverability and stability of articulated-linked buses. In works [7, 8] modular kinematic modeling of articulated buses was carried out and the kinematics of the N-trailer with off-axis interconnections were studied. Mathematical modeling of multi-link road trains movement was studied by authors of works [9, 10]. The works [10, 11] are devoted to the study of the stability and maneuverability of the road trains movement with the layout scheme "tractor car - two-axle trolley - three-axle semi-trailer".

The authors of work [16] investigated the influence of operational parameters and design of a coupling device on the dynamic interaction of the links during acceleration and movement on an uneven road of a light-duty road trains consisting of a 2-axle tractor and a single-axle trailer equipped with a braking system. The patterns of changes in indicators and the features of modeling the dynamic interaction of links have been established, and a mathematical model of motion has been developed.

Benjamin Esterl and Torsten Butz, when modeling a road train in [17], reduced obtained equations of motion to systems of ordinary differential equations. When considering a coupling device, additional restrictions are imposed on the relative movement of the truck and trailer. Real-time simulation results are demonstrated, proving the feasibility of the model for different types of coupling devices and complex driving maneuvers.

Kurtz E. F. in his work [18] conducted a scientific and technical review of the current state in the field of towing trailers by passenger cars, with a broader emphasis on "car-trailer" systems. The review covers both towing trailers by passenger cars and broader technical aspects of the interaction in the "car-trailer" system, namely the influence of aerodynamic forces on driving stability and the influence of transverse tire stiffness on driving dynamics.

Alejandro Bravo-Doddoli with Luis C. García-Naranjo in [19] obtained reduced equations of motion and gave the detailed description of dynamics for articulated vehicle N-trailer which moving under own inertia.

Dynamic simulation of vehicle control with n-trailers was performed in the work of Ali Keymasi Khalaji and Seyed Amir Ali Moosavian [20].

Xiang Kang and Wei Deng conducted a technical review of current state of the R&D area of vehicle and trailer handling dynamics and stability control [21]. Approaches to modeling road trains are systematized, including linear and nonlinear models, simulation methods and stability control. Special attention is paid to balancing the stability and controllability of the vehicle-trailer system.

In an experimental study [22] of the influence of trailer parameters (inertia, mass distribution, axle position) on the stability of a road train at high speeds - Jonathan Darling, David Tilley and Bo Gao found that mass distribution and axle position have a significant effect on stability, while a total mass of the trailer is less critical.

Mattia Zanchetta, Davide Tavernini and other authors in [23] investigated the torque vectoring formula to control of the yaw rate and the clutch angle of an articulated vehicle using the direct yaw moment which generated by towing vehicle.

Today, there are many designs of multi-link road trains. In our opinion, the layout scheme with a three-axle tractor, a 2-axle undercarriage and a 3-axle semi-trailer (Figs. 1 and 2) is the most universal, which follows from the analysis of the mentioned works.

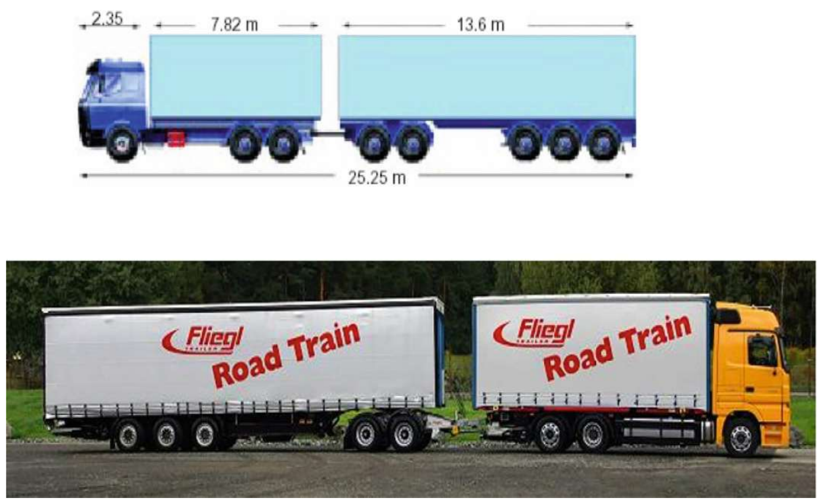

Fig. 1. Layout diagram of the road train with three-axle tractor, two-axle trolley "Dolly" and a three-axle semi-trailer [10].

Fig. 2. 2-axle trolley "Dolly" with fixed axles (trolley trailer) from "KRONE" company [10].

If a road train has 3 links or more, the study of movement of such multi-links road train is significantly complicated. This is due to the fact that we must take into account the simultaneous influence of a large number of factors on the nature of the road train links movement. During the movement of a road train, adjacent links receive dynamic force interaction, which significantly affects the movement of its individual links and can lead to significant deviations from the trajectory of movement set by the tractor truck (leading link).

Since road trains are means of increased danger, when designing and researching them, it is first necessary to pay significant attention to the study of theoretical patterns of their movement and behavior. The results obtained will be the basis for predicting their behavior in real conditions and will provide answers to a number of questions of organizational, technical and also legal nature.

3 Research Methodology

Based on the above, the layout of the road train with 3-axle tractor truck, 2-axle trolley and a 3-axle semi-trailer as universal and promising (developed by Scania & Krone) was chosen to study its maneuverability, which, first of all, affects traffic safety.

Considering that studies of the road trains movement of the specified layout scheme have practically not been conducted before, the purpose of our research is to develop the mathematical model of the modular three-link road train movement which consist from "three-axle tractor - two-axle trolley with non-adjustable axles - three-axle semi-trailer". In our opinion, the development of such mathematical model will allow us to conduct further research into the influence of layout and operational factors on the maneuverability of three-link truck trains to predict their behavior in real road conditions.

In this work, we will develop the new mathematical model of the movement for modular three-link trailer train consisting of a "tractor vehicle - two-axle dolly with non-rotating axles - three-axle semi-trailer", which would allow us to determine in theoretical studies the influence of the layout scheme and driving modes on its operational properties.

By introducing the forces of interaction of each wheel with the road into the mathematical model, it is possible to model different modes of movement of road trains (traction, free and braking), and it is also possible to influence the redistribution of masses along the axes and between the sides of the road train links. When developing a flat mathematical model of the road train movement of a separate layout scheme, the results of the works of Lobas L. [12, 13].

Figure 3 shows a calculation diagram of a three-link road train which consisted from three-axle tractor, two-axle trolley "Dolly" with non-rotating axles, and a 3-axle semi-trailer with all non-rotating axles when driving on an unstable curve.

The following symbols are used in the diagram and further in the text:

- Θ_0, Θ_2 – turning angles of the tractor steered wheels and the semi-trailer pivot axle, respectively;
- υ – course angle of the link and ω – angular velocity of the link;
- φ_1, φ_2 – folding angles of the trolley and semi-trailer respectively;
- C, m, I – center of mass, mass, central moment of inertia for road train link relative to the vertical axis;

- x, y – abscissa and ordinate for point C in the coordinate system O_{xyz};
- v, u - longitudinal and lateral velocity for the link center of mass;
- X_i, Y_i - longitudinal & lateral force on the wheels for road train corresponding link.

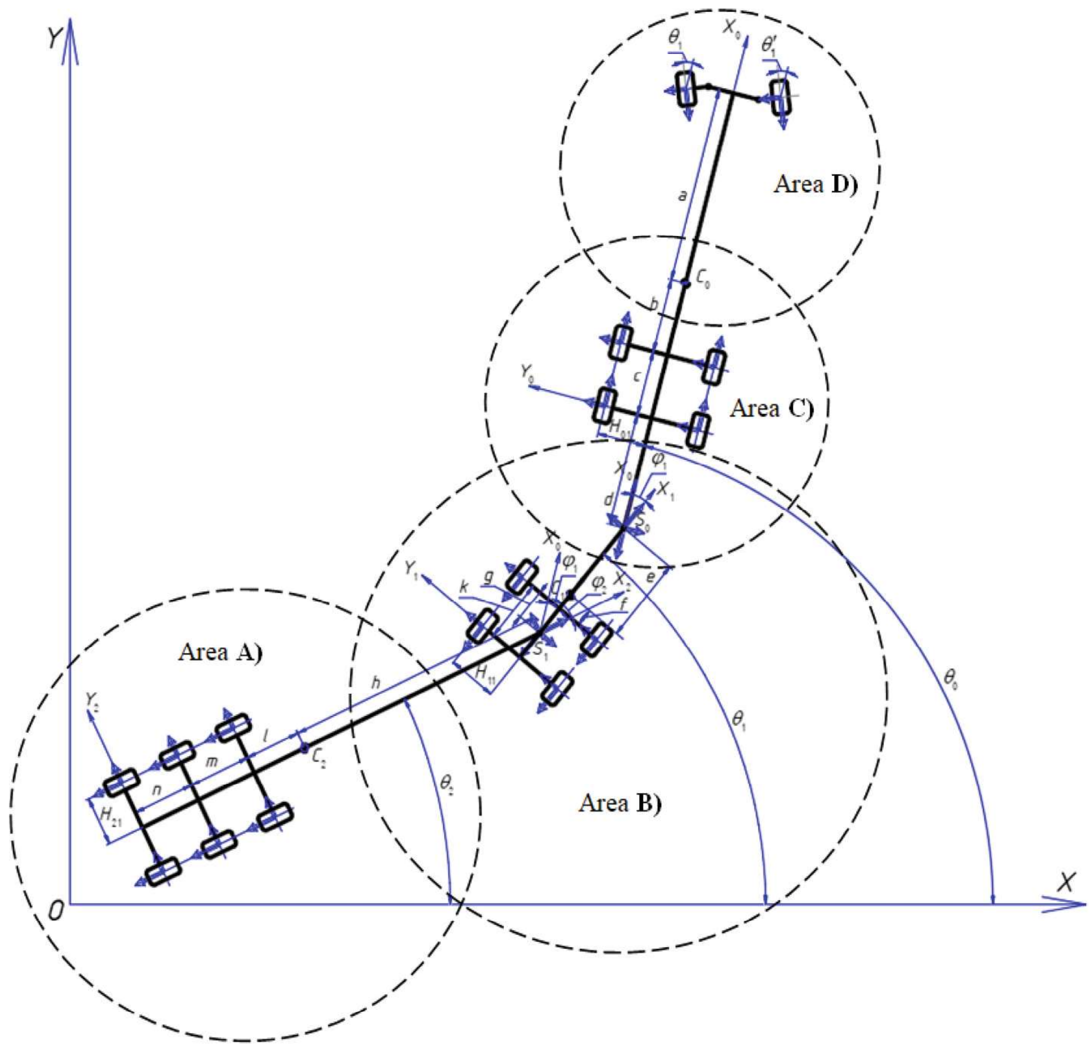

Fig. 3. Calculation scheme for 3-link road train when moving on an unstable turn.

The geometric parameters are clear from the figure. Indices: 0, 1, 2 – refer to the tractor, trolley and semi-trailer, respectively. Parameters marked with a "stroke" refer to the wheels on the starboard side of a road train, without the "stroke" mark – to the wheels on the port side.

The terms and definitions used in the work regarding the stability of road vehicles are applied in accordance with the State Standard of Ukraine 3736:98 [14].

The stability of road vehicles is determined in accordance with the Methods for determining the main parameters by testing of the State Standard of Ukraine 3310:96 [15].

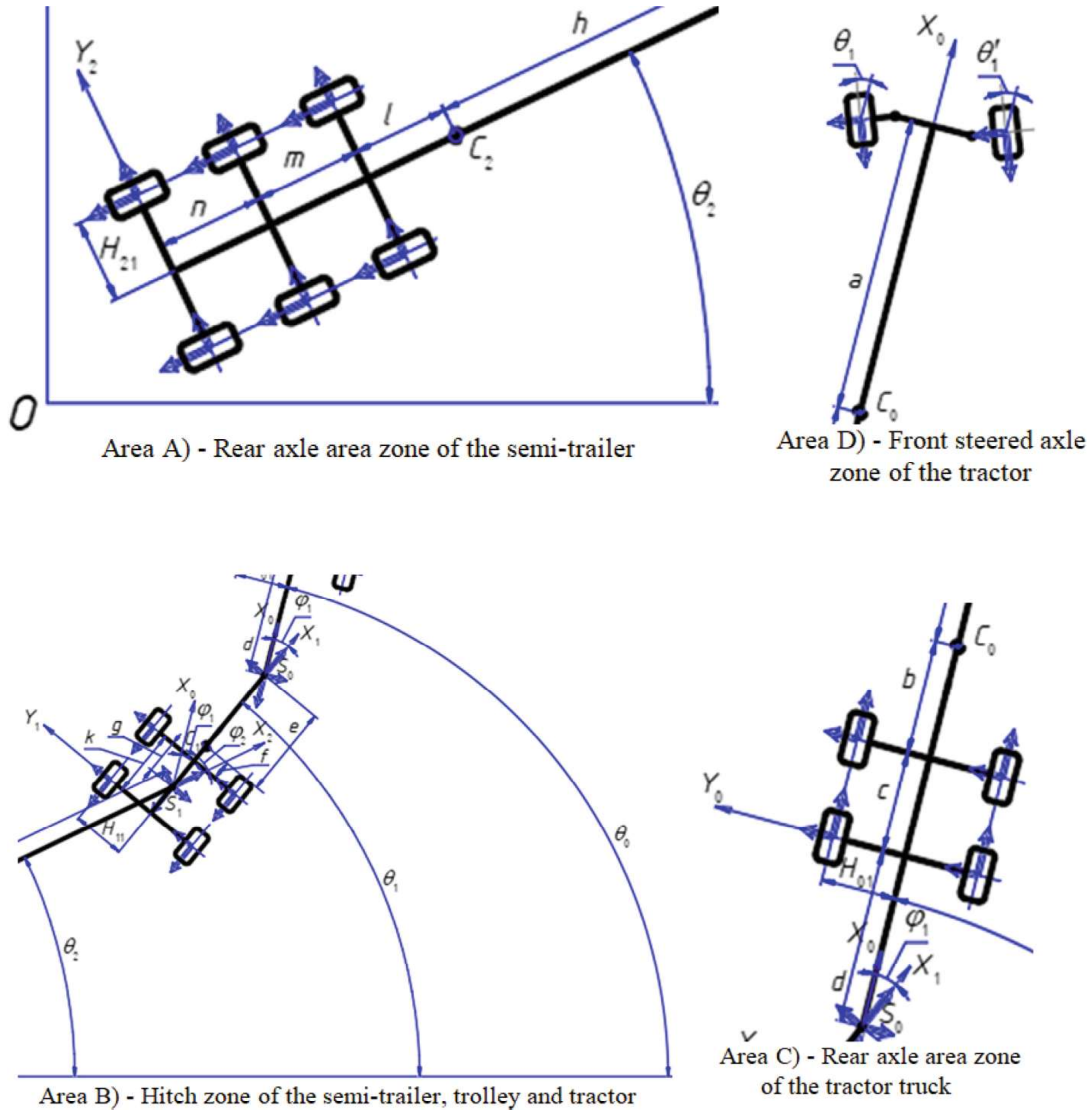

Fig. 4. Areas of calculation scheme for three-link road train.

4 Results and Discussion

In the diagram shown in Figs. 3, the road train performs plane-parallel motion. It is known that such motion of a rigid body consists of translational motion, in which all points of the body move with the speed of the pole, and rotational motion around this pole. In this case, the angular velocity does not depend on the choice of the pole.

To carry out a mathematical description of the plane-parallel motion of a truck-trailer combination with the possibility of taking into account changes that occur in the chassis and braking system during operation, we will introduce a number of simplifications and assumptions. We will assume that:

- the tractor, trolley and the semi-trailer move on a flat, absolutely solid and undeformed horizontal surface;
- there is no longitudinal or transverse slope of the surface;

- the normal reactions of the supporting surface act at the center of the wheel contact patch with the road surface and are equal in magnitude to the vertical load on the wheel;
- the main trajectory of the road train is a tractor vehicle center of mass trajectory;
- there are no friction, gaps and elastic elements in the supporting and coupling devices;
- there is no play in the steering drive hinges;
- gyroscopic moments and unbalanced moments of rotating parts are not taken into account;
- the frames of the tractor, trolley and semi-trailer are solid bodies with a longitudinal axis of symmetry, they have no torsional and bending deformations;
- the coordinates of the centers of unsprung and sprung masses are determined in the position of static equilibrium of the road train links;
- the elastic suspension elements of the links of a road train will be considered massless, they are characterized only by compliance, and their masses will be attributed to the sprung and unsprung masses of the links;
- the damping properties of the suspension and tires of the road train links are not taken into account;
- the position of the cargo on the car and semi-trailer will be considered such that it ensures the location of the centers of mass of the links in a vertical plane of their symmetry.

The speed of C_0 point during plane-parallel movement of the tractor along the moving base i_0, j_0 (orths of axes x_0, y_0) is determined by the formula:

$$v_{C_0} = i_0 v + j_0 u.$$

Components of the velocity for the trolley center of mass:

$$v_1 = v_0 \cos\varphi_1 - (u_0 - \omega_0(b+c+d))\ \sin\varphi_1,$$

$$u_1 = v_0\ \sin\varphi_1 + (u_0 - \omega_0(b+c+d))\ \cos\varphi_1 - \omega_1(k+f+e) \tag{1}$$

where $\omega_0 = \dot{\vartheta}_0$, $\omega_1 = \dot{\vartheta}_1 = \omega_0 - \dot{\varphi}_1$

The velocity components for the semi-trailer center of mass are written in the form:

$$\begin{aligned} v_2 &= v_0 \cos(\phi_2 + \phi_1) - (u_0 - \omega_0(b+c+d)) \sin(\phi_2 + \phi_1) + \omega_1(k+f+e-g) \sin\phi_2 \\ u_2 &= v_0 \sin(\phi_2 + \phi_1) + (u_0 - \omega_0(b+c+d)) \cos(\phi_2 + \phi_1) - \omega_1(k+f+e-g) \cos\phi_2 - \omega_2 h \end{aligned} \tag{2}$$

where $\omega_2 = \dot{\vartheta}_2 = \omega_1 - \dot{\varphi}_2$.

The quantities v, u, ω, φ_1, φ_2 can be determined from the dynamic equations of the road train motion, the center of mass coordinates and the angle course of tractor – using kinematic equations:

$$\begin{cases} \dot{x} = v_0 \cos\vartheta_0 - u_0 \sin\vartheta_0 \\ \dot{y} = v_0 \sin\vartheta_0 + u_0 \cos\vartheta_0 \\ \dot{\vartheta}_0 = \omega_0 \end{cases} \tag{3}$$

The quasi-coordinate method was used to compile a differential equations of a road train motion. The equation of motion is composed with respect to the values v, u, ω – the parameters of the tractor's motion based on the theorem on the change of a forces main vector $\vec{K}$ and a main moment of momentum $\vec{G}$, which are expressed through kinetic energy T [9]. The transition to quasi-velocities allows us to separate the variables in a given dynamical system into two subsystems of lower dimension, which are integrated sequentially.

$$\begin{cases} K = grad_{\vec{v}} T \\ G = grad_{\vec{\omega}} T \end{cases} \quad \begin{cases} K_i = \dfrac{\delta T}{\delta v_i} \\ G_i = \dfrac{\delta T}{\delta \omega_i} \end{cases} \quad (i = 1, 2, 3) \tag{4}$$

where $\vec{v}$ - absolute speed, $\vec{\omega}$ - absolute angular velocity.

The law of change of momentum and kinetic moment in a moving coordinate system can be written as:

$$\begin{cases} \frac{d\vec{K}}{dt} + \vec{\omega} \cdot K = F; \\ \frac{dG}{dt} + \vec{\omega} \cdot G + \vec{v} \cdot K = M \end{cases}. \tag{5}$$

After projecting these equations onto the moving coordinate axes, taking into account the plane-parallel motion of the road train, we obtain:

$$\begin{cases} \frac{d}{dt}\frac{\delta T}{\delta v_0} - \omega_0 \frac{\delta T}{\delta u_0} = F_{x_0} \\ \frac{d}{dt}\frac{\delta T}{\delta u_0} + \omega_0 \frac{\delta T}{\delta v_0} = F_{y_0} \\ \frac{d}{dt}\frac{\delta T}{\delta \omega_0} + v_0 \frac{\delta T}{\delta u_0} - u_0 \frac{\delta T}{\delta v_0} = M_{\vartheta_0} \end{cases} \tag{6}$$

To determine the folding angles of the road-train trailer links φ_1, φ_2, which are holonomic coordinates, it is sufficient to write the second kind Lagrange equation:

$$\begin{cases} \dfrac{d}{dt}\dfrac{\delta T}{\delta \dot{\varphi}_1} - \dfrac{\delta T}{\delta \varphi_1} = M_{\varphi_1} \\ \dfrac{d}{dt}\dfrac{\delta T}{\delta \dot{\varphi}_2} - \dfrac{\delta T}{\delta \varphi_2} = M_{\varphi_2} \end{cases} \tag{7}$$

According to the well-known recommendations [12, 13] after replacing the third equation in system (6) and the first in system (7) with linear combinations of equations for ω, φ_1, φ_2, we will obtain the system of equations which describing a motion of road train in general form:

$$\begin{cases} \frac{d}{dt}\frac{\delta T}{\delta v_0} - \omega_0 \frac{\delta T}{\delta u_0} = F_{x_0} \\ \frac{d}{dt}\frac{\delta T}{\delta u_0} + \omega_0 \frac{\delta T}{\delta v_0} = F_{y_0} \\ \frac{d}{dt}\left(\frac{\delta T}{\delta \omega_0} + \frac{\delta T}{\delta \dot{\varphi}_1}\right) + v_0 \frac{\delta T}{\delta u_0} - u_0 \frac{\delta T}{\delta v_0} - \frac{\delta T}{\delta \varphi_1} = M_{\vartheta_0} + M_{\varphi_1} \\ \frac{d}{dt}\left(\frac{\delta T}{\delta \dot{\varphi}_1} - \frac{\delta T}{\delta \dot{\varphi}_2}\right) - \frac{\delta T}{\delta \varphi_1} - \frac{\delta T}{\delta \varphi_2} = M_{\varphi_1} - M_{\varphi_2} \\ \frac{d}{dt}\frac{\delta T}{\delta \dot{\varphi}_2} - \frac{\delta T}{\delta \varphi_2} = M_{\varphi_2} \end{cases} \tag{8}$$

here F_{x_0}, F_{y_0} - projections of the main vector $\vec{F}$ on the moving axis x_0 and y_0 respectively, M_{ϑ_0}, M_{φ_1}, M_{φ_2} - generalized moments by the corresponding coordinates: $M_{\vartheta_0} = mom_{C_0 z_0} F$, $M_{\varphi_1} = -mom_{S_0 z_1}(F_1 + F_2)$, $M_{\varphi_2} = -mom_{S_1 z_2} F_2$.

It is possible to obtain expressions for the moments and forces acting on the links of a road train if we consider their corresponding diagrams. Figure 4 shows a diagram of forces acting on the axles of a tractor and trolley. Used symbols are similar to shown on Fig. 3, and the geometric parameters are clear from the figure.

On the Fig. 4, the trolley is conventionally disconnected from the tractor, and their force interaction is reflected by reactions in the traction coupling (Fig. 5).

The expressions for the moments and forces acting on the road train links can be written in the form:

$$\begin{aligned}
F_{x_0} = {} & -X_{01}\cos\theta_0 - Y_0\sin\theta_0 - X'_{01}\cos\theta'_0 - Y'_{01}\sin\theta'_0 \\
& -\left(X_{02} + X'_{02}\right) - \left(X_{03} + X'_{03}\right) \\
& -\left(X_{11} + X'_{11}\right)\cos\varphi_1 + \left(Y_{11} + Y'_{11}\right)\sin\varphi_1 \\
& -\left(X_{12} + X'_{12}\right)\cos\varphi_1 + \left(Y_{12} + Y'_{12}\right)\sin\varphi_1 \\
& -\left(X_{21} + X'_{21}\right)\cos(\varphi_1 + \varphi_2) + \left(Y_{21} + Y'_{21}\right)\sin(\varphi_1 + \varphi_2) \\
& -(X_{22} + X'_{22})\cos(\varphi_1 + \varphi_2) + (Y_{22} + Y'_{22})\sin(\varphi_1 + \varphi_2) \\
& -(X_{23} + X'_{23})\cos(\varphi_1 + \varphi_2) + (Y_{23} + Y'_{23})\sin(\varphi_1 + \varphi_2);
\end{aligned}$$

$$\begin{aligned}
F_{y_0} = {} & Y_{01}\cos\theta_0 - X_{01}\sin\theta_0 + Y'_{01}\cos\theta'_0 - X'_{01}\sin\theta'_0 \\
& +\left(Y_{02} + Y'_{02}\right) + \left(Y_{03} + Y'_{03}\right) \\
& +\left(Y_{11} + Y'_{11}\right)\cos\varphi_1 + \left(X_{11} + X'_{11}\right)\sin\varphi_1 \\
& +\left(Y_{12} + Y'_{12}\right)\cos\varphi_1 + \left(X_{12} + X'_{12}\right)\sin\varphi_1 \\
& +\left(Y_{21} + Y'_{21}\right)\cos(\varphi_1 + \varphi_2) + \left(X_{21} + X'_{21}\right)\sin(\varphi_1 + \varphi_2) \\
& +(Y_{22} + Y'_{22})\cos(\varphi_1 + \varphi_2) + (X_{22} + X'_{22})\sin(\varphi_1 + \varphi_2) \\
& +(Y_{23} + Y'_{23})\cos(\varphi_1 + \varphi_2) + (X_{23} + X'_{23})\sin(\varphi_1 + \varphi_2);
\end{aligned}$$

$$M_{\vartheta_0} + M_{\varphi_1} = \left(\begin{array}{l}
\left(X_{01} - X'_{01}\right)\varepsilon_0 + \left(X_{01}\cos\theta_0 - X'_{01}\cos\theta'_0 + Y_{01}\sin\theta_0 - Y'_0\sin\theta'_0\right)H_{01} + \\
+\left(Y_{01}\cos\theta_0 - X_{01}\sin\theta_0 + Y'_{01}\cos\theta'_0 - X'_{01}\sin\theta'_0\right)a + \\
+\left(X_{02} - X'_{02} + X_{03} - X'_{03}\right)H_{02} - \left(Y_{02} + Y'_{02}\right)b - \left(Y_{03} + Y'_{03}\right)(b + c) + \\
+(b + c + d)\left(\begin{array}{l}
\left(Y_{11} + Y'_{11}\right)\cos\varphi_1 + \left(X_{11} + X'_{11}\right)\sin\varphi_1 + \\
\left(Y_{12} + Y'_{12}\right)\cos\varphi_1 + \left(X_{12} + X'_{12}\right)\sin\varphi_1 + \\
+\left(Y_{21} + Y'_{21}\right)\cos(\varphi_1 + \varphi_2) + \left(X_{21} + X'_{21}\right)\sin(\varphi_1 + \varphi_2) + \\
+(Y_{22} + Y'_{22})\cos(\varphi_1 + \varphi_2) + (X_{22} + X'_{22})\sin(\varphi_1 + \varphi_2) + \\
+(Y_{23} + Y'_{23})\cos(\varphi_1 + \varphi_2) + (X_{23} + X'_{23})\sin(\varphi_1 + \varphi_2)
\end{array}\right)
\end{array}\right)$$

$$M_{\varphi_1} - M_{\varphi_2} = -\left(\begin{array}{l}
\left(X_{11} - X'_{11}\right)H_{11} - \left(Y_{11} + Y'_{11}\right)(f + e) + \\
\left(X_{12} - X'_{12}\right)H_{11} - \left(Y_{12} + Y'_{12}\right)(k + f + e) + \\
\\
+e\left(\begin{array}{l}
+\left(Y_{21} + Y'_{21}\right)\cos\varphi_2 + \left(X_{21} + X'_{21}\right)\sin\varphi_2 + \\
+(Y_{22} + Y'_{22})\cos\varphi_2 + (X_{22} + X'_{22})\sin\varphi_2 + \\
+(Y_{23} + Y'_{23})\cos\varphi_2 + (X_{23} + X'_{23})\sin\varphi_2
\end{array}\right)
\end{array}\right);$$

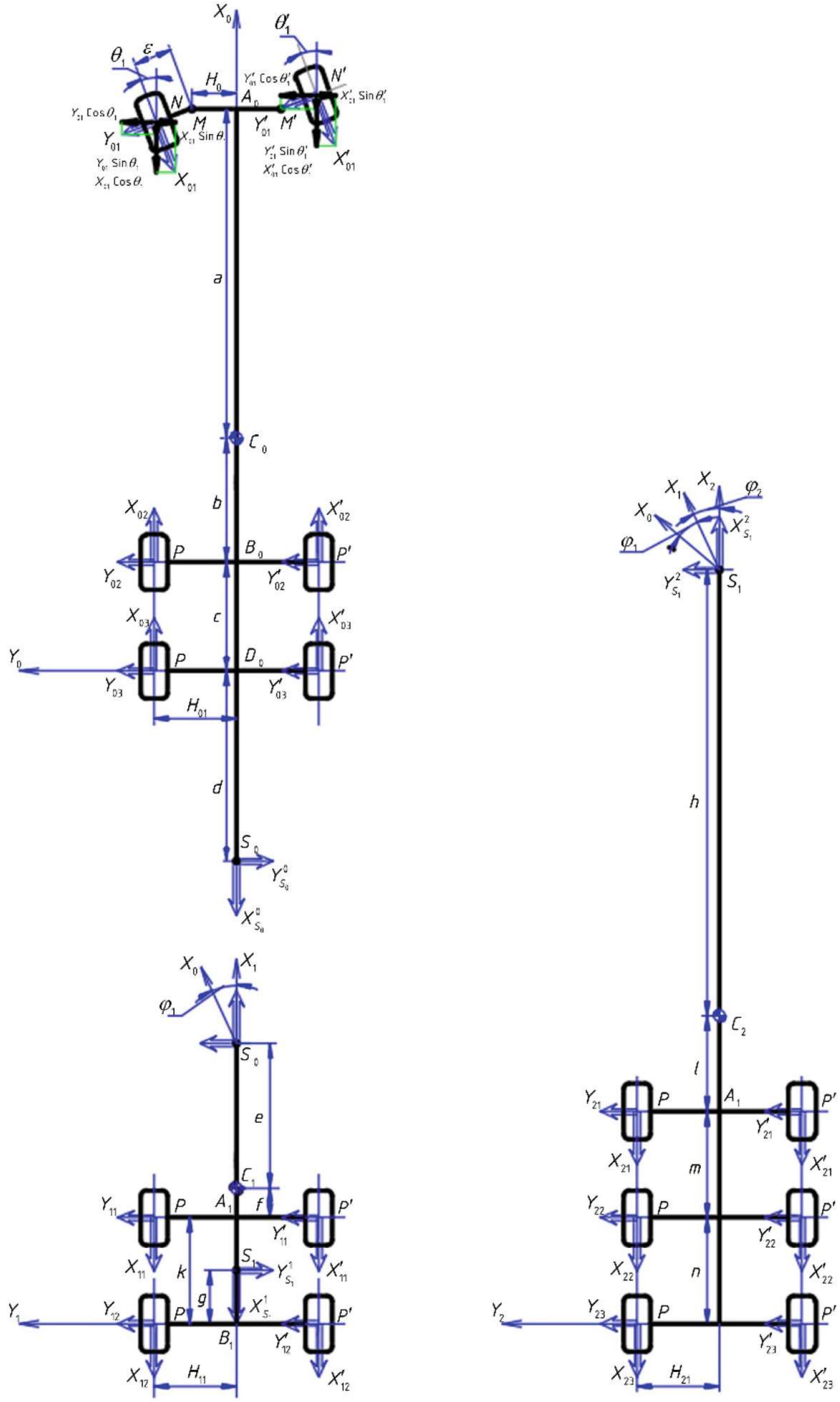

Fig. 5. Diagram of forces acting on the axles of the tractor, trolley and the semi-trailer.

$$M_{\varphi_2} = -\begin{pmatrix} \left(X_{21} - X'_{21}\right)H_{21} - \left(Y_{21} + Y'_{21}\right)(h + l) + \left(X_{21} - X'_{21}\right)\varepsilon_{21} + \\ +\left(X_{22} - X'_{22}\right)H_{21} - \left(Y_{22} + Y'_{22}\right)(h + l + m) + \left(X_{22} - X'_{22}\right)\varepsilon_{22} + \\ +\left(X_{23} - X'_{23}\right)H_{21} - \left(Y_{23} + Y'_{23}\right)(h + l + m + n) + \left(X_{23} - X'_{23}\right)\varepsilon_{23} \end{pmatrix}. \tag{9}$$

Kinetic energy for a road train can be represented as the sum of the kinetic energies of its links:

$$T = \sum T_i \tag{10}$$

here T_i - kinetic energy of a road train link, can be determined by the known equation

$$T_i = \frac{1}{2}\left(m_i V_{C_i}^2 + I_i \dot{\vartheta}_i^2\right) \tag{11}$$

For a 3-link road train, Eq. (10) taking into account Eq. (11) can be written as:

$$T = \frac{1}{2}\left(m_0 V_{C_0}^2 + I_0 \dot{\vartheta}_0^2 + m_1 V_{C_1}^2 + I_1 \dot{\vartheta}_1^2 + m_2 V_{C_2}^2 + I_2 \dot{\vartheta}_2^2\right) \tag{12}$$

Taking into account Eqs. (1) and (2), Eq. (12) is transformed to the form:

$$T = \frac{1}{2}\begin{bmatrix} m_0\left(v_0^2 + u_0^2\right) + I_0\dot{\vartheta}_0^2 + \\ +m_1\begin{pmatrix} (v_0 \cos\varphi_1 - (u_0 - \omega_0(b + c + d))\ \sin\varphi_1)^2 + \\ +(v_0\ \sin\varphi_1 + (u_0 - \omega_0(b + c + d))\ \cos\varphi_1 - \omega_1(f + e))^2 \end{pmatrix} + \\ +I_1\dot{\vartheta}_1^2 + \\ +m_2\begin{pmatrix} \begin{pmatrix} v_0 \cos(\varphi_2 + \varphi_1) - (u_0 - \omega_0(b + c + d))\sin(\varphi_2 + \varphi_1) + \\ +\omega_1(k + f + e - g)\sin\varphi_2 \end{pmatrix}^2 + \\ +\begin{pmatrix} v_0 \sin(\varphi_2 + \varphi_1) + (u_0 - \omega_0(b + c + d))\cos(\varphi_2 + \varphi_1) - \\ -\omega_1(k + f + e - g)\cos\varphi_2 - \omega_2 h \end{pmatrix}^2 \end{pmatrix} + I_2\dot{\vartheta}_2^2 \end{bmatrix} \tag{13}$$

From Eqs. (8), (9) and (13) we obtain expressions for determining the variables characterizing the motion of the tractor vehicle (components of linear velocity v and u, angular velocity ω) and the trailer links of the road train (folding angles of the undercarriage φ_1 and the semi-trailer φ_2).

These expressions together with the system of Eqs. (3) constitute a mathematical model of the 3-link road train motion for the selected layout scheme. The differential equations of a road train motion in general form for the 3-link road train is written by the system of differential Eqs. (14):

$$\begin{cases} m_0(\dot{v}_0 - \omega_0 u_0) = F_{x0} \\ m_0(\dot{u}_0 + \omega_0 v_0) = F_{y0} \\ I_0\dot{\omega}_0 = M_{c0} \\ \frac{d}{dt}\left(\frac{\delta T}{\delta\dot{\varphi}_1} - \frac{\delta T}{\delta\dot{\varphi}_2}\right) - \frac{\delta T}{\delta\varphi_1} + \frac{\delta T}{\delta\varphi_2} = M_{\varphi_1} - M_{\varphi_2} \\ \frac{d}{dt}\frac{\delta T}{\delta\dot{\varphi}_2} - \frac{\delta T}{\delta\varphi_2} = M_{\varphi_2} \end{cases} \tag{14}$$

here: F_{x0},F_{y0} – the projection of a main vector of external forces acting on a tractor vehicle, respectively on its longitudinal and transverse axes; M_{c0} – main moment of external forces relative to tractor vehicle center of mass; $M_{\varphi_1}, M_{\varphi_2}$ – generalized moments by the corresponding coordinates.

So, in our research:

- the specifics of multi-link road trains of the specified layout scheme, their weight, load distribution and dynamic behavior are taken into account;
- a model based on a Lagrangian approach of the second kind has been developed. This allows us to take into account the wide range of dynamic interactions between system components and significantly increase the accuracy of predicting its behavior.

Taking into account the design features of a proposed scheme of the 3-link road train makes the model practically applicable in engineering practice for optimizing the designs of similar road trains used in the real sector of the economy, agricultural industry or in transport area. The obtained analytical dependencies can be integrated into software for engineering analysis of motion dynamics. The theoretical results obtained in our work have good correlation with conducted research the dynamic interaction of road train links [16].

5 Conclusions

The contact force interaction of adjacent links during the multi-link vehicle movement is dynamically extends to the entire road train and leads to significant deviations of its links from the direction of movement set by the tractor truck (leading link).

The layout scheme of a road train with 3-axle tractor, 2-axle trolley and 3-axle semi-trailer as universal and promising was chosen to study its movement and develop the mathematical model of movement.

This work develops the new mathematical model of the movement for modular three-link trailer truck consisting of a "tractor truck - two-axle trolley with fixed axles (dolly) - three-axle semi-trailer".

As part of the work performed, a number of important scientific and applied results were obtained, which are important both for further research in the field of vehicle dynamics and for practical application in automotive engineering.

A complex mathematical model of the road train movement with a 3-axle tractor, 2-axle trolley and 3-axle semi-trailer has been created, taking into account the specific location of the coupling point outside the tractor base, which allows for accurate reproduction of the dynamic interaction between the elements of the "tractor-trolley-semi-trailer" system.

The model was not validated using specialized software or experimental data, but its accuracy was assessed by analyzing its compliance with physical laws and theoretical expectations. The analysis of the correctness of the results was carried out by comparing them with real physical processes – in particular, the regularities of the vehicle's movement, its reaction to input parameters, and compliance with the expected dynamic properties (for example, stability of movement, acceleration, etc.) were checked. Model assumptions and theoretical estimates were also used, which allowed us to draw conclusions about the adequacy of the model within the accepted assumptions.

The available model is designed for a 3-axle tractor, 2-axle trolley and 3-axle semi-trailer. However, it can be expanded or narrowed for other Layout schemes, but this will require minor changes and changes to the mathematical description in accordance with the new configuration. This scheme is suitable for further optimization of the kinematic and force parameters of the structure.

The modeling results provide a basis for developing active safety systems and improving existing vehicle control methods. The proposed model can be used as a basis for creating virtual test benches or integration into software for engineering analysis of motion dynamics.

Further research will be aimed at improving the obtained mathematical model and checking the consistency of the obtained theoretical data with experimental studies of three-link road train in real road conditions.

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