Influence of the City Transport Route Network Discrete Model Geometrical Parameters on a Quality of a Passenger Traffic System Operation

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Abstract. The research is dedicated to an investigation of an influence of geometrical parameters of a discrete model of the city transport route network on a quality of a passenger traffic system operation and to a discovery of the priority criteria of the developed characteristics while searching for the ways of improvement of its structural components. Quality characteristics of the city passenger routes network substantially depend on a complexity of their geometrical configuration and topological structure. Afterwards, a development of a city routes network goes, as a rule, by the way of complication of a travel routes topological structure and of interrelation between geometrical characteristics of their separate elements. Such a trend allows to make a conclusion about the necessity of elaboration of the effective mathematical methods of modeling of the new ones and of optimization of the existing networks, on the base of which there would be situated the algorithms of the quantity evaluation of a quality of the city transport system operation on the base of an all-round analysis of geometrical parameters of the route schemes discrete models. In the present research work there were examined and analyzed the discrete models of geometrical images, their main characteristics, there were offered the methods of determining of separate parameters of the geometrical structures concerning the determining of optimal ways of improvement of the technical and technological characteristics of a city transport network. All the examined images can be set as principles of analysis of its accessibility and determining of the possibilities and ways of improvement of the already existing routes network.

Keywords: Transport route network · Field of fractal type · Discrete cell model · Transport accessibility

1 Introduction

While solving the practical problems concerning an analysis and identification of some objects on the pictures, the actual task is to determine certain fields that correspond to the

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informative characteristics of given structures (schemes, maps, photos of the surfaces, photos of the materials and some their combinations).

For summarizing of the necessary information about the objects on the pictures and its application for solving some specific tasks, it is necessary to have a methodology and the algorithms of calculating of the main characteristics of the objects on the pictures: the parameters of their form, a geometrical structure, an objects composition, other characteristics. Basing on the analysis of the determined characteristics, there are generalized some conclusions about the objects types, the examined picture quality concerning the determining of the ways of improvement of its technical and technological characteristics [1]. Along with that it is clear that the base of analysis is composed by exactly the geometrical characteristics either of its entire picture, or of its separate fragments.

Second aspect of the problem is that a majority of pictures is, on practice, the totalities of the complex, "chaotic-cut" objects and they are identified as fractal or quasifractal images. It's exactly a fractal dimension (together with other geometrical characteristics) that can become a base for evaluation and effective improvement of characteristics initially of a model, but then of real objects respectively [2].

2 Literature Review

There exist a lot of researches dedicated to an analysis and treatment of the pictures [3–5]. Among them there can be separated the researches in a field of alarm systems, quality control systems of different goods, the searches of identification of the objects and text documents, the scientific studies concerning medical diagnostics and other.

In the before-mentioned studies there are solved, as a rule, three classic problems concerning graphical pictures: a problem of picture synthesis, that is to get it by a description, a problem of analysis, that is to get a description by a picture, a problem of picture treatment, that is to get a new picture basing on the existing one. Computer technique and information technologies are actively used while solving all these problems. But the problems of developing of the algorithms of effective presentation and analysis of the pictures that contain the objects of evidently fractal type, are still actual. In the researches [6, 7] an effectiveness of the methods of fractal geometry to determine the main technological characteristics and to predict the ways of optimisation of the objects that have fractal structure, is shown. The methods of discrete presentation of fractal objects, the manners of identification, the algorithms of their dimension calculating, the use of complex geometrical characteristics to work out the ways of improvement of examined images are the actual problems both in theoretical and practical aspects.

3 Research Methodology

The geometrical characteristics of the objects form the basis of solving the majority of practical problems. If these characteristics are calculated and analyzed on the base of pictures of objects and sets, it is necessary to determine their list, the methods of calculating and priority in the measurement order. Mostly, the picture fragment measured parameters can be the next: an identification of an object type, its topological dimension, the linear dimensions, a perimeter, an area, the form parameters (convexity, concentricity, compactness, roundness), the static moments of closed areas and other characteristics.

Because of the discreteness of a picture of, for example, an examined city transport network (see Fig. 1a), two variants of calculating its geometrical characteristics are possible: contour and “skeletal”. At contour method every point of a picture is treated as a discrete element (can be a pixel) and has a dimension equal to its area correspondingly. Linear distances between nearby points are equal to a unit of a scale of discrete presentation. While using “skeletal” method of calculating the geometrical characteristics, it is considered that a point is situated in the centre of sampling element. Thus, a calculating of vertical and horizontal distances between separate points is made by the linear dimensions of sampling element, the diagonal ones are determined as a hypotenuse of a right triangle.

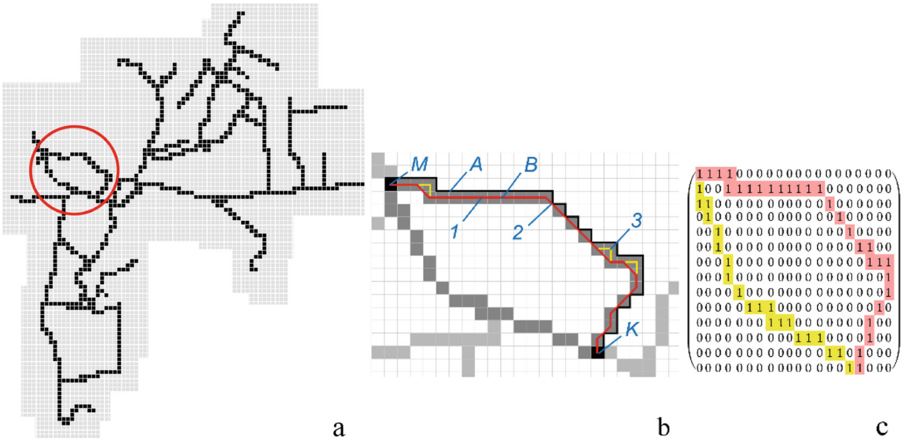

Fig. 1. Discrete pixel model of a city routes network, the scheme of “skeletalization” of the fragment and its matrix implementation. *A* – segment length on the base of contour method. *B* – segment length on the base of skeletal method. *M*, *K* – end points of curvilinear discrete fragment of a curve.

Both methods are right and on the short steps of sampling give the results that have slight, that is admissible, error. That is why, while solving practical problems, one can choose more algorithmically convenient way. But except calculating the geometrical characteristics, a created discrete model of a picture should predict a maximum informational content and a possibility of reverse passage to its continuous presentation. That is exactly a “skeletal” method of calculating of linear dimensions that mostly corresponds to the requirements of our formulated practical problem. On Fig. 1b there are given the discrete pixel models of geometrical elements of network and there are offered the schemes of their “skeletization”.

One can notice that the offered discrete cell models of the unidimensional images have an ambiguous “skeletization” (1 – right, 2 – diagonal, 3 – mixed). Within a solving of our practical problem, while choosing a form of “skeleton” on an area of discretely presented image, a diagonal connexion type of nearby points would have a priority.

Let's show an effectiveness of such choice while modelling and identifying some geometrical images. For discrete model of a curve segment the important characteristics are: a topological (fractal) dimension of an object, a degree of rectilinearity and its length. A topological dimension is determined by the extreme (end) points of a discrete model and by calculating a fractal dimension. The model limits are the zero-dimensional images. A fractal dimension of a segment in a process of scaling approaches to 1, that is – an object is unidimensional, and an area of an object model is equal to zero.

A degree of rectilinearity of separate segments is determined by a relation of calculated parameter of length of a discrete model of segment by a "skeletal" method to a distance between the end points of an object. A length of a discrete model of a transport network segment is determined by a matrix of "skeleton" (see Fig. 1c), built with taking into consideration the types of coherence of separate discrete elements.

According to a model, N_L is the cells with direct neighbours (neighbourhood – line), N_P is the cells with indirect neighbours (neighbourhood – point). While having for one cell both vertical and horizontal neighbours with a limit – line, two connections are replaced by the diagonal one (see Fig. 1b).

A model length while using a "skeletal" method is calculated according to a formula:

$$L_M = unit(n \cdot Ng_L + k \cdot N_T), \tag{1}$$

where Ng_L is a horizontal distance between the centres of nearby cells of a model; Nv_L is a vertical distance between the centres of nearby cells of a model; N_T is a diagonal distance between the centers of nearby cells of a model; *unit* is the scale parameters of an elementary discrete cell; *n* and *k* is a number of horizontal and vertical elements of skeleton correspondingly.

A length between the end points of an object is calculated:

$$L_{go} = \sqrt{(unit(Ng - 1))^2 + (unit(Nv - 1))^2}, \tag{2}$$

where *Ng* is a number of the model cells of a segment in horizontal direction; *Nv* is a number of the model cells of a segment in vertical direction.

For rectilinear segments, situated horizontally, vertically and at an angle 45° – a degree of rectilinearity of a discrete model W_{lin} would be always equal to 1. In all other cases $W_{lin} \approx 1.01 = 1.05$.

For curvilinear discrete model (see Fig. 1b) the basic geometrical characteristics would be: a topological (fractal) dimension of an object, a degree of curvilinearity, a model perimeter, the parameters of a form of closed curves and other characteristics.

A topological dimension is determined by the end points (for open curves) of a discrete model and by a calculating of fractal dimension of an element. The limits of a discrete model fragment are the zero-dimensional images (points *M* and *K*). A fractal dimension of an object, in a process of scaling, approaches to 1, that is – an object is unidimensional.

A matrix model of a transport network segment is shown at Fig. 1c. A length of a curvilinear section *M-K* is determined according to an Eq. (1).

A degree of curvilinearity of a section *M-K* is determined by a relation of a calculated parameter of a model section length $L_{M\text{-}K}$ to a distance between the end points of an object *M* and *K*.

If $L_{go} = 0$ – curve line is closed. Then, instead of a length of curve section according to a "skeletal" method, there is calculated a perimeter of a closed curve model, for example $P_{M\text{-}M}$. In general view a perimeter is presented as a function $P_{M-M} = f(Ng_L, Nv_L, N_T)$. As $L_{go} = 0$ – there is no sense to calculate a degree of curvilinearity for closed curve.

The parameters of a closed curve model form (convexity, concentricity, compactness and roundness) would have more geometrical informational content in such a case. To determine the listed geometrical characteristics of a binary picture of a closed curve model, it is necessary, first of all, to build on its end points a limitary rectangular shell (see Fig. 2), and secondary to determine a cell that would present a weight centre (i_{cg}, j_{cg}) of a closed field of a curve discrete model.

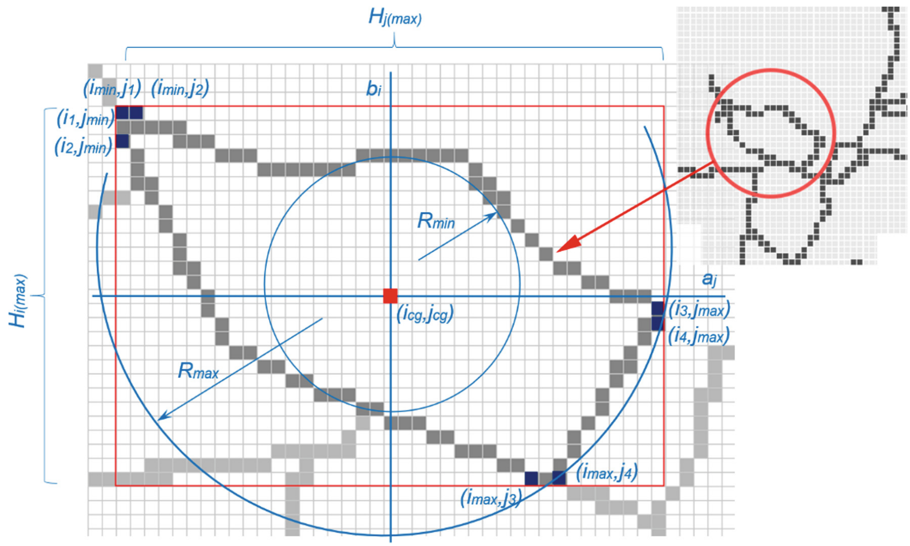

Fig. 2. Scheme of calculating the geometrical parameters of a city transport network fragment in binary reference.

The discrete coordinates of a field weight centre are calculated by the formulas:

$$i_{cg} = \frac{\sum\limits_{(i,j)\in(R-Q)} i}{\sum\limits_{(i,j)\in R} unit \cdot n - \sum\limits_{(i,j)\in Q} unit \cdot k}, j_{cg} = \frac{\sum\limits_{(i,j)\in(R-Q)} j}{\sum\limits_{(i,j)\in R} unit \cdot n - \sum\limits_{(i,j)\in Q} unit \cdot k}, \quad (3)$$

where a dominator of the expressions (3) is an area of inner field of a given model. If a value of desired discrete quantities i_{cg}, or j_{cg} is non-integral, one should use the rules of mathematic rounding to integral values.

An area is determined as an amount of all elementary cells of a model of closed field R, except for the limiting cells – set Q. If a scaled cell coefficient is taken for unit, one would have a formula:

$$S_{in} = \sum_{(i,j)\in R} unit \cdot n - \sum_{(i,j)\in Q} unit \cdot k. \quad (4)$$

Afterwards, a degree of convexity C_1 of a discrete pixel model of a closed curve would be determined by a proximity of its full area R to an area of limiting rectangular shell – $Hi(max) \times Hj(max)$

$$C_1 = \frac{\sum_{(i,j)\in R} unit \cdot n}{(i(\max) \times j(\max))unit}. \quad (5)$$

The closer the given index is to 1, the bigger is a degree of convexity of a discrete model. As a second form parameter would be considered a degree of concentricity C_2 of a discrete model of closed curve. It is determined by a relation of the radiuses, inscribed R_{min} and circumscribed R_{max} from a weight centre, of the circles (see Fig. 2).

$$C_2 = \frac{R_{\min}}{R_{\max}} = \frac{\min\sqrt{(i_k - i_{cg})^2 + (j_k - j_{cg})^2}}{\max\sqrt{(i_k - i_{cg})^2 + (j_k - j_{cg})^2}}. \quad (6)$$

An important geometrical parameter is a degree of compactness C_3. A circle has the biggest compactness. That is why a proximity of the areas of a closed curve discrete model and a circle with the same perimeter would be determined by C_3. Respectively, a perimeter and an area of a figure are equal $P_f = f(Ng_L, Nv_L, N_T)$, $S_f = \sum_{(i,j)\in R} unit \cdot n$.

An area of a circle with a perimeter P_f is calculated:

$$S_k = \frac{P_f^2}{4\pi}. \quad (7)$$

A degree of a model compactness, taking into consideration (7), looks like:

$$C_3 = \frac{4\pi \sum_{(i,j)\in R} unit \cdot n}{P_f^2}. \quad (8)$$

A degree of roundness of a closed curve discrete model is a very interesting index from a geometrical point of view. It can be calculated by two ways that gives almost equal results in a majority of cases. A parameter is rather sensible to a cut, oscillating behavior of a limit of discrete curve model and can immediately, without using the fractal algorithms, classify the objects of fractal or non-fractal type.

The first way is based on a calculating of relation of a perimeter square of given field to its area:

$$C_4 = \frac{P_f^2}{S_f}. \quad (9)$$

The second way consists in determination of relation of a middle deviation of the discrete image limiting cells from a field weight centre to a middle square deviation of the same cells from a weight centre:

$$C_5 = \frac{\frac{1}{K}\sum_{0}^{K-1}\left|(i_k, j_k) - (i_{cg}, j_{cg})\right|}{\sqrt{\frac{1}{K}\sum_{0}^{K-1}((i_k, j_k) - (i_{cg}, j_{cg}))^2}}. \tag{10}$$

The bigger value have the parameters of roundness C_4 or C_5 of a curve discrete model, the bigger is the possibility to predict a fractal behavior of an examined field.

A limiting rectangular (see Fig. 2), that restrains the maximum and minimum values *i* and *j* of the black cells of a discrete model, determines, together with a weight centre, the basic axes of a closed geometrical image that also influence the form characteristics and play an important role while determining a model orientation in space.

The next elements of the images models identification are the closed fields of non-fractal and fractal types. If a discrete cell model of a picture or of picture fragment has at least one black cell that has 8 black neighbors by 8-coherence, one can consider that a given model presents a two-dimensional non-fractal image, or a field with a dimension from 1 to 2 of fractal type.

The limits of such object are a unidimensional set of cells of fractal or non-fractal type with at minimum two black neighbors by 8-coherence. These limiting cells are in the base of developing a “skeleton” of a field limit.

For a discrete model of two-dimensional image the basic geometrical characteristics are: a topological (fractal) dimension of an object, a perimeter value, an area of object model, a convexity, a concentricity, a compactness, a roundness, the static moments, Euler characteristic etc.

A topological dimension is determined inductively by a dimension of the limits of a discrete model and by a calculating of its fractal dimension. As it was before-mentioned, a limit of a discrete model (for non-fractal images) is a unidimensional set of cells with a determined “skeleton”, respectively, a topological dimension of an object is equal to 2. A fractal dimension of a discrete model, while scaling, for a cell calculating method, also approaches to 2.

A perimeter of a closed field discrete model. This value is considered as a contour characteristic of two-dimensional images, that is why a “skeletal” calculating method is applied to it. A perimeter is determined taking into account all the cell types of an image limit Nv_L, Ng_L, N_T.

A distance between the vertical and horizontal neighbors is equal to unit, a distance between the diagonal ones – to $unit \cdot \sqrt{2}$. While having for one cell of a “skeleton” simultaneously both vertical and horizontal neighbors with a number of pairs – *s*, two connections Nv_L and Ng_L are replaced by one diagonal N_T.

Afterwards, a perimeter of a limiting line of a cell model of two-dimensional image is calculated by a length of a “skeleton”:

$$P_M = ((n - s) \cdot Ng_L + (m - s) \cdot Nv_L + (k + s) \cdot N_T) \cdot unit, \tag{11}$$

where *n* is a total number of horizontal linear connections Ng_L between the cells of a model limit; *m* is a total number of vertical linear connections Nv_L between the cells of a model limit; k is a total number of diagonal point connections N_T between the cells of a model limit.

The static moments have a wide application at a process of analysis and geometrical identification of the pictures of closed fields. While having a matrix picture model, it is easy to determine the main of them.

Thus, a zero moment $\mu_{00} = S_f$ is calculated as an amount of elementary cells of inner field of an object model and completely corresponds to its area. The moments of first order that are normalized to this area, $\mu_{01} = i_{cg}, \mu_{10} = j_{cg}$ as it is before-mentioned, are equal to the coordinates of a field weight centre.

The central moments of first order that are normalized to a field area, can give an information about a form roundness of a field limiting contour, and central moments of second order show how symmetrical the field is correspondingly to the horizontal and vertical axes:

$$\mu_{11} = \frac{\sum\limits_{(i,j)\in R-Q} (i - i_{cg})(j - j_{cg})}{S_f}, \mu_{02} = \frac{\sum\limits_{(i,j)\in R-Q} (i - i_{cg})^2}{S_f}, \mu_{20} = \frac{\sum\limits_{(i,j)\in R-Q} (j - j_{cg})^2}{S_f}. \quad (12)$$

Two-dimensional fields and their discrete cell structures can have even more complicated structure by its topology (combined sets, enclosure, “nicks” etc.). A complexity of such structures can be evaluated by Euler number that characterises an inclusion of the fields in each other, a number of openings or “nicks” inside a given combined object.

4 Results

The before examined and analyzed discrete models of geometrical images, their main characteristics, the offered methods of determination of separate parameters of geometrical structures are closely linked with a concrete practical problem – a determination of optimal ways of improvement of technical and technological characteristics of a city transport network [8–10].

According to a discrete model of a city transport network (see Fig. 1a), there are separated the next geometrical elements that compose a picture base: the models of right and unclosed curve lines, the models of closed curves, the discrete models of the fields of non-fractal and fractal types, the binary models of combined structures. All the examined images are important for identification of the network elements and can be set as principals of analysis of its accessibility and for determination of the possibilities and ways of improvement of the existing routes net. Thus, for example, the diametral, radial and tangential routes of passenger transport are described by the discrete models of the unclosed curve lines, the shortest distances between the transport junctions are the straight line models, the circle routes are presented as the closed images of fractal and non-fractal types. The pictures of separate transport city districts are modeled as the closed curvilinear fields, and a superposition of route schemes in separate transport junctions are described by the discrete models of combined structures [11, 12].

An offered identification method of the picture elements can become a base for an analysis of an existing state of the city route network, but the only identification is not enough to elaborate the offers concerning the routes improvement. That is why for every element of route network there are determined the basic geometrical characteristics, there are given the criteria of priority of their influence on a routes network quality, there are elaborated the algorithms of calculation of the basic parameters for a discrete model of the routes schemes. Inside a set of geometrical characteristics as both a whole network and its separate components, there are distinguished: a topological (fractal) dimension, the linear dimensions, a perimeter, an area, the parameters of a discrete image form (a convexity, a concentricity, a compactness, a roundness), the static moments of the closed fields and other that indirectly influence the technical and technological characteristics of a route network. For example: the linear dimensions, an image perimeter are a measure of routes extension, of distances between the stop points, of distances between the basic transport junctions; an area of closed fields is a basic value to determine and to correct the form parameters, that, in their way, are the determinant for a maximum transport accessibility of separate points and territories on a city map; the static moments together with an index of fractal dimension, characterize a compactness of a route schemes network on a determined territory; the Euler numbers give an information about a "critical layout", that is an inaccessibility of some city zones for transport service [13–15].

As a result of conducted researches, there was offered a hypothesis that a city is considered optimal by a transport accessibility if an amount of the chains lengths that connect all the cells of a discrete cell model of a city, is minimal (see Fig. 3). Such an optimal network model has a fractal structure, and its separate components are considered transport accessible, if: for each of them there exist a possibility to get in any other cell of a discrete city map model; the total extension of the routes, and correspondingly a time of their passage, are minimal; a number of changes to reach a desirable purpose is minimal; a maximum number of routes goes through a determined point.

Fig. 3. Hypothesis of a transport accessibility model.

Let's show an effectiveness of an offered methodology at an example of determination of the indices of transport accessibility in Lutsk city as a whole and the possible ways of its improvement. For this purpose let's impose on a city map an existing transport network of the buses and trolleybuses routes and let's make its binarization. Let's make a sampling of a received map taking into account the conditions of distance of any point not more than 500 m to a route network, and let's determine a number of cells that cover it (in this case there are 354 of them, Fig. 4a).

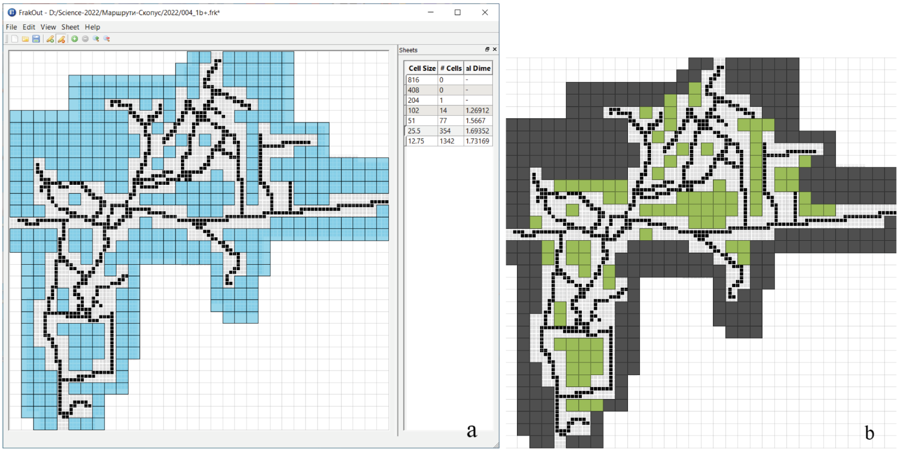

Fig. 4. Graphical presentation of an optimization methodology of the city routes network according to its accessibility: (*a*) algorithm of determination of fractal dimension of a city routes network; (*b*) correction of the city routes schemes basing on the analysis of fractal dimension.

Next, let's determine the zones on a discrete map model that are completely inaccessible for transport service while satisfying the standardized requirements. Such "dead" zones are coloured grey (see Fig. 4b), a total number of the cells that describe them, is equal to 267. With the help of special algorithms one can calculate a fractal dimension of a city map discrete model provided that an indice of transport accessibility is equal to 0. A fractal dimension of such a model is equal to 1.84.

Let's calculate a fractal dimension in a condition of an existing route network of Lutsk city. It is equal to 1.69.

A conducted fractal analysis clearly shows a tendency to improvement of a transport accessibility in a city – it's a decrease of a fractal dimension of presented model. A practical application of given idea and calculations comes to a correction of the city route schemes to liquidate the green zones of an offered model, with a purpose to greatly decrease its fractal dimension, that is – an improvement of the indices of a city transport network accessibility in general (see Fig. 4b). A model with a dimension close to 1 would be characterized by an ideal transport accessibility.

Besides, according to the before-mentioned algorithms, it is possible to determine the quality indices of transport sufficiency for given model: a total extension of a route into the necessary point, an approximate time of travel by a route, a possible number of changes to reach a purpose, a number of possible variants of the vehicles choice on a route to get to a point of destination and other.

Though, while critically evaluating the results of made researches, it is necessary to mention that the offered fractal methodology of searching the ways of improvement of a city transport accessibility is only one of the possible approaches of determination of the optimal ways of improvement of technical and technological characteristics of a transport network, but not a final solution of a problem of optimization of transport accessibility.

5 Conclusion

In the present research work there was examined an influence of geometrical parameters of a discrete model of the city transport route network on a quality of a passenger traffic system operation and there were circumscribed the criteria of priority of taking into consideration the geometrical characteristics while searching the ways of improvement of the transport network structure components. There were elaborated the methods of pictures identification and there were distinguished separate geometrical elements of a discrete model of the city transport network that significally influence a quality of a passenger traffic.

There were elaborated the algorithms and the methods of calculating the geometrical characteristics of picture identification for determination and further effective correction of technical and technological characteristics of the city transport structure.

References

1. Ablamejko, S.V., Nedz'ved', A.M.: Processing of Optical Images of Cellular Structures in Medicine. OIPI NAN Belarusi (2005)
2. Feder, J.: Fractals. Springer, Boston (1988). https://doi.org/10.1007/978-1-4899-2124-6_4
3. Mohan, V.M., Kanaka Durga, R., Devathi, S., Srujan Raju, K.: Image Processing Representation Using Binary Image; Grayscale, Color Image, and Histogram. In: Satapathy, S.C., Raju, K.S., Mandal, J.K., Bhateja, V. (eds.) Proceedings of the Second International Conference on Computer and Communication Technologies. AISC, vol. 381, pp. 353–361. Springer, New Delhi (2016). https://doi.org/10.1007/978-81-322-2526-3_37
4. Serino, L.: Skeletonization of Gray-Tone Images Based on Region Analysis. In: Sanfeliu, A., Martínez Trinidad, J.F., Carrasco Ochoa, J.A. (eds.) CIARP 2004. LNCS, vol. 3287, pp. 495–502. Springer, Heidelberg (2004). https://doi.org/10.1007/978-3-540-30463-0_62
5. Marszałek, M., Schmid, C.: Accurate object Recognition with Shape Masks. Int. J. Comput. Vision **97**, 191–209 (2012). https://doi.org/10.1007/s11263-011-0479-2
6. Pustiulha, S., et al.: Fractal diagnostics of the degree of fuel atomization by diesel engine injectors. Eastern-European Journal of Enterprise Technologies **6**(8), 40–46 (2017). https://doi.org/10.15587/1729-4061.2017.116104
7. Pustiulha, S., Holovachuk, I., Samchuk, V., Samostian, V., Prydiuk, V.: Improvement of the Technology of Tribostate Application of Powder Paints Using Fractal Analysis of Spray Quality. In: Ivanov, V., et al. (eds.) DSMIE 2019. LNME, pp. 280–289. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-22365-6_28
8. Dubovik, V.O.: Methods for assessing the transport accessibility of the territory. Reg. Stud. **4**(42), 11–18 (2013)

9. Bonenberg, W., Bonenberg, A., Dong, L., Zhou, M.: Transport Accessibility as a Factor of Spatial Development on the Example of the Poznan Metropolitan Area, Poland. In: Charytonowicz, J., Maciejko, A., Falcão, C.S. (eds.) AHFE 2021. LNNS, vol. 272, pp. 294–301. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-80710-8_36
10. Ribeiro, J., Fontes, T., Soares, C., Borges, J.L.: Accessibility as an indicator to estimate social exclusion in public transport. Transportation Research Procedia **52**, 740–747 (2021). https://doi.org/10.1016/j.trpro.2021.01.019
11. Renzi, A., Ferranti, P.: Efficient and nice – urban accessibility and public transport. In: Quality of Life in Urban Landscapes. UBS. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-65581-9_12
12. Horák, J., Ivan, I., Voženílek, V., Tesla, J.: Multidimensional Evaluation of Public Transport Accessibility. In: Ivan, I., Horák, J., Inspektor, T. (eds.) GIS OSTRAVA 2017. LNGC, pp. 149–164. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-61297-3_11
13. Fuglsang, M., Hansen, H.S., Münier, B.: Accessibility Analysis and Modelling in Public Transport Networks – A Raster Based Approach. In: Murgante, B., Gervasi, O., Iglesias, A., Taniar, D., Apduhan, B.O. (eds.) ICCSA 2011. LNCS, vol. 6782, pp. 207–224. Springer, Heidelberg (2011). https://doi.org/10.1007/978-3-642-21928-3_15
14. Cavallaro, F., Dianin, A.: An innovative model to estimate the accessibility of a destination by public transport. Transp. Res. Part D: Transp. Environ. **80**, 102256 (2020). https://doi.org/10.1016/j.trd.2020.102256
15. Benenson, I., Ben-Elia, E., Rofe, Y., Rosental, A.: Estimation of Urban Transport Accessibility at the Spatial Resolution of an Individual Traveler. In: Thakuriah, P., Tilahun, N., Zellner, M. (eds.) Seeing Cities Through Big Data. SG, pp. 383–404. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-40902-3_21