

Mathematical Modeling Formation the Waviness of the Bearings Parts in Centerless Mortise Grinding on Rigid Supports

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Abstract. The formation of corrugation of the working surfaces of bearing parts is associated with fluctuations in the size of the cut layer of metal and changes in the components of the cutting force. Laplace operators were used to modeling the centerless grinding system, on the basis of which the transfer function and the characteristic equation were constructed. It was found that the formation of waviness depends on the position of the hodograph of the movement of the vector of the center of the part in the complex plane, which in turn depends on the geometric parameters of the rigid supports of the centerless grinder machine. This makes it possible on the basis of hodographs and the angular orientation of their asymptotes to determine the geometric stability of the process depending on the angles of adjustment of the rigid supports of the grinder machine. To confirm the correctness of the hypotheses, two methodological approaches were used: multiplication of hodographs of wave and regeneration displacement and coincidence of the combined hodograph of regeneration and waviness displacement mechanisms with the hodograph of infinitely rigid machine displacement. The diagrams which allow choosing geometry of adjustment of rigid support that allows to increase or decrease parameters of certain harmonics are developed. The three-dimensional diagram allows you to set the local minima, which are characterized by acceptable geometric conditions of adjustment, providing regulated waviness of the working surfaces of bearing parts.

Keywords: Adjustment, Asymptote, Centerless Grinding, Harmonica, Hodograph, Roller Bearing, Support.

1 Introduction

During the manufacture of detail of roller bearings, the quality of treated rolling surfaces is important [1]. When grinding the functional rolling surfaces of roller bearing rings, it is necessary to ensure rapid removal of allowance and high processing performance, the final macro-and micro geometric accuracy of the treated surface, com-

pliance with physical and mechanical requirements [2]. The size of the cut layer during grinding is proportional to the normal grinding force [3]. The formation of the waviness of the details is caused by deviations in the size of the cut layer or changes in the grinding force [4]. Surface roughness depends on the material of the part and the grinding wheel, the productivity of the allowance removal, and the relative speed between the part and the grinding wheel. The main variable for the analysis of the machining process is the grinding force [5].

The scheme of the dynamic model, which was used to predict both static (geometric) and dynamic stability of the process of centerless grinding on rigid supports is shown in Fig. 1.

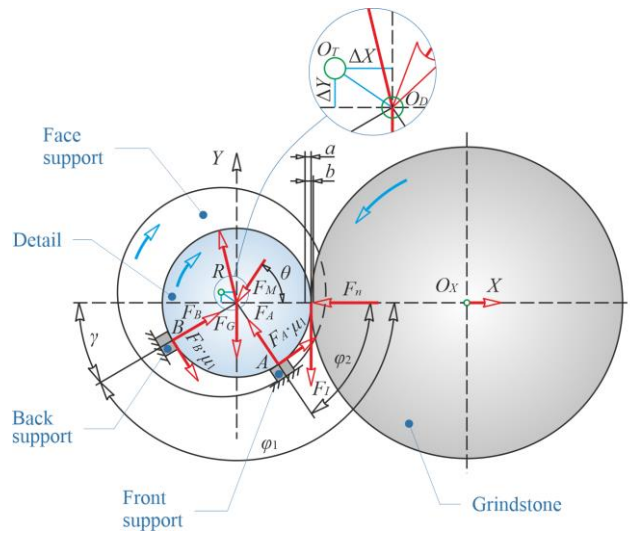


Fig. 1. Scheme of a dynamic model for predicting the stability of the centerless grinding

During the study of the centerless grinding process, the tangential force vectors of supports A and B were not taken into account, since they have small values and do not significantly affect the modeling. The components of the grinding forces F_n , F_t taking into account the force F_G arising from the weight of the part and the force F_M from magnetic retention, form the formation of force vectors of the reactions of the supports F_A , F_B which is necessary to ensure the static balance of the grinding system.

As can be seen from Fig. 1 the center of the part is shifted relative to the center of the support by a small amount (ΔX ; ΔY). This is done to ensure permanent contact with the supports of the part. One of the main parameters of the centerless grinding process that can be controlled is the angles of inclination of the supports (φ_1 and φ_2).

That is why production requires recommendations for choosing the value of the angles of the supports and thanks to this obtaining the final regulated harmonics.

However, such recommendations require research and scientific justification of their results [6].

2 Literature Review

To research the grinding processes, there are mathematical models used by Laplace operators to analyze the centerless grinding system [7]. The research of the transfer function and the solution of the characteristic equation is carried out using graph-analytical methods. The formation of waviness depends on the position of the hodograph of the movement of the vector of the center of the detail in the complex plane, which in turn depends on the geometric parameters of the rigid supports of the grinder machine [8]. Adjustment angles φ_1 or φ_2 have a direct effect on the normal component of grinding force, depth of cutting, stability in the cutting zone, and, as a consequence, on the harmonics of the newly formed waviness of bearing details [9].

In the Laplace region, multiplication is required to combine the mechanism of waviness and the mechanism of regeneration [10]. For graphical analysis, the mechanism of the combined effect is determined by multiplying the vector by the corresponding value of $s = \sigma + jn$. In this case, the coordinate origin vector for both ripple and regeneration mechanisms is at the same frequency and σ (here, $\sigma = 0$). In vector multiplication, the values of each vector are multiplied and angles from the positive real axis are added. For example, Fig. 2, and shows the multiplication of the vector at a frequency of $n = 7.5$. The waveform has a magnitude of 2 and an angle of -45° , and the regeneration vector has a magnitude of 3 and an angle of 120° . The combined vector for this condition is shown in Fig. 2, b where the value of $n = 7.5$ is the vector 6 (2, 3) and the angle is 75° ($120^\circ + (-45^\circ)$). The part of the hodograph of the harmonic vector 6 to 8 shows the general picture of the combined hodograph of the vector for different harmonics and conditions set up [11].

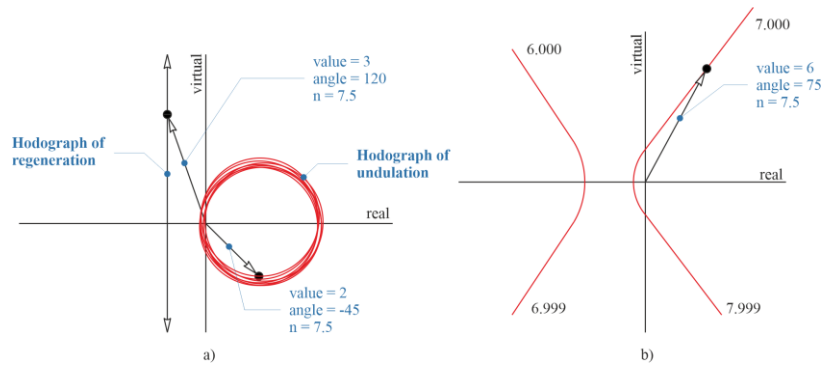


Fig. 2. Vector multiplication scheme (a) and combined hodograph of the vector from 6 to 8 harmonics (b)

Elements of the sample of the combined operation of the hodograph mechanism from the 2nd to the 10th harmonic under a specific debugging condition [12, 13]. These

forms of hodograph hyperbola are concentrated around a point $(-0.5; 0)$ in the complex plane and have infinite asymptotes covering the range from 0° to 360° (Fig. 3).

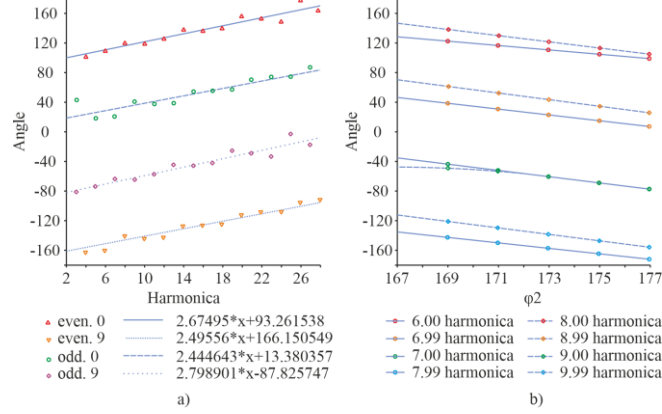


Fig. 3. Tendencies of the asymptotic angle of the combined displacement hodograph when changing the harmonic (a) and when changing the angle of the rear support (b)

Asymptotes have imaginary components, starting with positive infinity for integers numbered harmonics (5,000, 6,000, etc.) and ending with negative infinity for numbers of smaller integers (5,999, 6,999, etc.). It is established that the most variable for these hodographs is the angular orientation of the asymptotes. This confirms that the asymptotes around 0° mean the geometric instability of the process.

3 Research Methodology

We will perform an analysis of the change in harmonics of waviness depending on the geometry of the setup of rigid supports (Fig. 3, a). In the asymptotes, there is a tendency to increase by 2.5° per harmonic with even and odd, grouping together at a value $\approx 180^\circ$ from each other. In Fig. 3, b shows the trends of the asymptotes of the angles with a change in the angle φ_2 for some harmonics if the angle of the front support is constant, $\varphi_1 = 55^\circ$. This graph shows a decrease in the asymptote of the angle if φ_2 is increased, it is also possible to observe even and odd, whole, and next-to-whole asymptotes of angles that are $\approx 180^\circ$ apart from each other. Since the asymptotes of angles are clear arrays, we need the interpretation of angles to explain the geometry of centerless grinding processes.

The study proposes to use two methodologies. The first is the procedure of multiplication of hodographs of waviness displacement and regeneration. The second is in the analysis of the graphic coincidence of the combined hodograph of regeneration and waviness movement mechanisms with the hodograph of infinitely rigid machine movement [14, 15]. The analysis showed that the displacement hodograph has a negative real part, so the mechanism of the waviness effect on the process is unstable [16]. At the same time, the angle of such hodograph movement of the waviness mechanism

will acquire a value greater than 90° or less than -90° from the positive real axis. Since the hodograph of the movement of the regeneration mechanism is in a certain area, the angle of the hodograph will be similar at a given frequency. If the waviness and regeneration mechanisms coincide at a certain frequency (geometric angles are compensated), the combined displacement hodograph forms a hyperbola with an asymptote of 0° (-360°). Such a compensation mechanism indicates the geometric instability of the asymptotes for some harmonics [17]. For an infinitely rigid machine, the hodograph of displacement in the complex plane is located at the point at the origin of the coordinates. At the same time, the hodograph of waviness movement and regeneration enters the area around the point $(-0.5; 0)$. The displacement hodographs, which have an asymptote near 0° , usually intersect with the machine displacement hodograph. The fact that the results fall in the system of the process mechanism corresponding to the interaction for this indicated frequency of the ripple allows the growth of this harmonic due to instability in the processing zone. The given interpretation involves the use of an infinitely rigid machine, which is why the study points to the geometric instability of the waviness frequency, which consists of a vector coincidence in the complex plane. It has been established that the asymptote angles largely depend on the number of harmonics and the geometry of tuning angles, so it allows predicting which harmonics will be unstable in a given range of tuning angles. If a certain angle of the asymptote of one harmonic is changed and investigated during the full complex of tuning geometry, it is observed that the theoretical geometric instability of the specified number of harmonics will be manifested if the angle of the asymptote is a multiple of the value in radians -2π . In the course of the research, a contour diagram of the asymptote angle was obtained, and 14 harmonics were studied within the standard geometry of centerless grinding on rigid supports (Fig. 4).

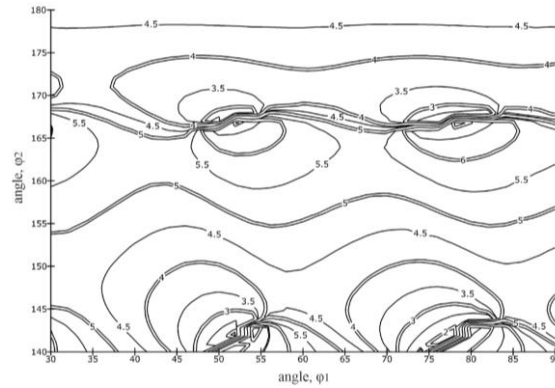


Fig. 4. Contour diagram of asymptote angles in radians for the 14th harmonic within the typical geometry of centerless grinding

The angular shift from 2π to 0 radians leads to a high density of contours allows you to easily see places where the asymptotes are multiples of 2π . This graph shows that the 14th harmonic is theoretically unstable at $\varphi_1 = 52^\circ, 78^\circ$ and $\varphi_2 = 143^\circ, 168^\circ$. Although only one line in each of these regions will be exactly 0° , instability ripples are

common for harmonics whose numbers are away from the whole due to a phase shift in the regeneration of waviness regeneration during grinding. Therefore, it is best to look only at the general region, where the whole harmonic is unstable. For example, in Fig. 4 you can draw an ellipse around each of the areas of the concentrated contour lines to represent the main zones where 14th harmonics are theoretically unstable.

During the research, results were obtained that made it possible to construct a diagram of the geometric instability of the waviness for a certain range of harmonics (2...22) for the standard conditions of the geometry of the grinding process adjustment based on the asymptote angles. The diagram confirms that the vast majority of combinations during adjustment of the angles of rigid supports are in the region of geometric instability and the instability of harmonics corresponds to a certain array of values. For example, consider the 12th harmonic, which is geometrically unstable in the region of $\varphi_1 = 60^\circ$ and $\varphi_2 = 165^\circ$, the 14th harmonic is geometrically unstable in the region above and to the left near the values of $\varphi_1 = 52^\circ$ and $\varphi_2 = 168^\circ$.

This diagram allows you to select the geometry of the adjustment of grinding on rigid supports, which increases or decreases the parameters of a particular harmonic.

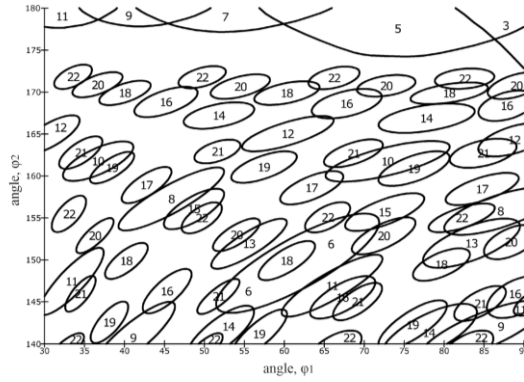


Fig. 5. The geometric instability wave diagram is constructed using angular analysis of the combined displacement hodograph at the stability limit within the typical debug geometry

Analysis of Fig. 5 makes it possible with high probability to predict the geometric instability in the cutting zone during grinding, thereby predicting the growth or reduction of a harmonic ripple.

4 Results

4.1 Impact diagram based on angular analysis of the hodograph

When the combined hodograph of the displacement and regeneration mechanisms for any harmonic had an asymptote at 0° , this harmonic was geometrically unstable. The angles of the asymptotes followed the main trends depending on the conditions of

adjustment of the machine. Given this fact, it is fair to assume that the closer the angle of the asymptote to 0° , the less stable the harmonic becomes.

The summary diagram is defined as an indicator of the degree of instability as a whole for each debugging condition. When performing this action, there is a need for a parameter that would be proportional to the angle of the asymptote, and angles around 0 or 2π should have a significant effect on it and angles far from 0 or 2π have little effect. The cosine function satisfies the established requirements. If we take the values of the cosines of the radians shown in Fig. 4, the results diagram suggests a relative level of instability for 14th harmonics under different debugging conditions. In this diagram (Fig. 6), contours with levels close to zero (0.1, 0.2) are much more stable than levels around 1.0.

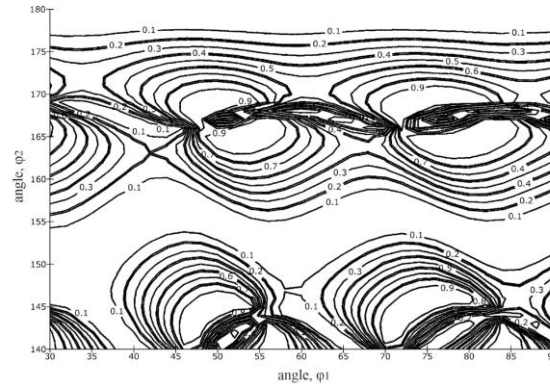


Fig. 6. Contour diagram of cosines of asymptote angles for the 14th harmonic within the typical geometry of centerless grinding

If the perform analysis is similar to Fig. 6 for each harmonic from 2 to 30 within the usual geometry of the centerless grinding, we obtain a general diagram of the results of geometric stability, which is shown in Fig. 7.

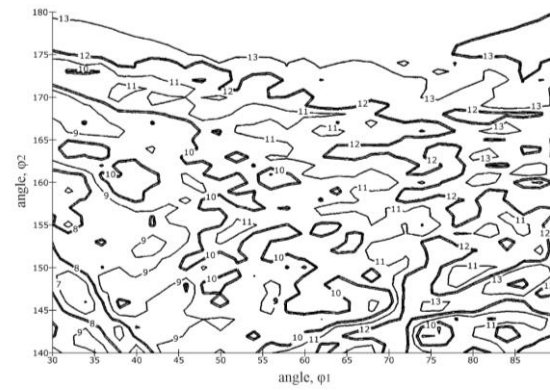


Fig. 7. Contour diagram of all cosines of asymptote angles from 2 to 30th harmonics within the typical geometry of centerless grinding

4.2 Nonzero sigma, impact diagram based on hodograph diameter analysis

If we analyze the combined hodograph of undulating displacement and regeneration with non-zero growth rate, σ , its graphical representation changes dramatically. Non-zero σ changes the hodograph of regeneration movement from a straight line to a circle. For a positive σ , the circles are centered closer to the point $(-1; 0)$, while the negative σ forms circles centered around the origin $(0; 0)$. The size of the hodograph responds faster than the exponential function, so that only at the slightest change in the value of σ does it significantly change the appearance of the combined hodograph of waviness displacement and regeneration.

Figure 8 shows a combined hodograph of waviness displacement and regeneration with different positive σ from 6,000 to 7,999 harmonics under a given debugging condition. Thus, the hyperbola rotates around itself forming cyclic spirals. As the sigma value increases, the size of the spirals decreases. With an absolute increase in the sigma, the size of the spirals decreases.

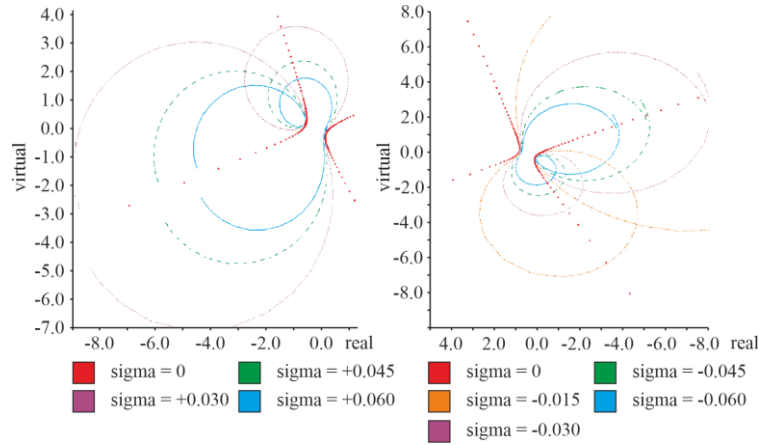


Fig. 8. Scheme of change of the combined hodograph of movement in a spiral at $+\sigma$ (a) and at $-\sigma$ (b)

An infinitely rigid machine in the complex plane is the point at the origin. When the hodograph of the movement of the waviness and regeneration intersects with the hodograph of the movement of this machine, the result is a geometric instability in the grinding zone. If a certain spiral intersects with the hodograph of the machine movement, it should be small enough to be near the origin. Therefore, any value of σ that can make the spiral small enough to intersect with the hodograph of the machine movement will form individual harmonics of unstable level. That is, if one helix is large compared to another at a certain σ , it is clear that a larger helix will need a larger σ to become small enough to intersect with the hodograph of machine movement. On the other hand, the best condition for grinding will be the result of the debugging condition, the spirals of which form a very small instability (small σ) intersecting with the hodograph of the machine movement. Based on this analysis, the debugging con-

dition with large spirals is less desirable than the debugging condition with smaller spirals at a certain unstable σ .

Using these features, an estimate of the geometric stability of the grinder is obtained, based on the size of the combined hodograph of waviness displacement and regeneration at constant, nonzero σ . If we consider in the complex plane of the spiral of all harmonics under all debugging conditions with some non-constant σ , the total size of the spirals will be proportional to the instability under each debugging condition. Figure 9 illustrates the sum of the total size of the spirals from 2 to 30th harmonics for each debugging condition within the normal geometry of debugging centerless grinding.

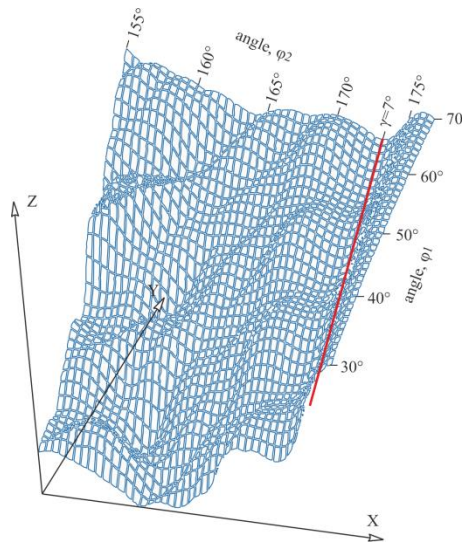


Fig. 9. Three-dimensional diagram of the total size of the combined displacement hodograph for 2...30th harmonics

A three-dimensional diagram is shown for conceptual interpretation of research results.

5 Conclusions

So, as a result of the conducted research, it was established that the characteristics of the waviness of the working surfaces of the roller bearing rings are influenced by the geometry of the setup of rigid supports during centerless grinding. It is shown that different support angles during processing have different effects on the harmonics of the waviness of the working surfaces of roller bearings.

The mechanism of waviness on the surface of the part in the process of centerless mortise grinding depends on a number of additional factors (characteristics of the

grinding wheel, machine vibration, grinding force, initial and time-varying waviness of the part), which must be considered to ensure stable quality.

The stability of the centerless grinding process is influenced by the geometry of the rigid supports. This is shown by the corresponding diagrams. In particular, on the influence diagram based on the angular analysis of the hodograph, provided that the system finds a stability limit $\sigma = 0$, lower values of the adjustment angles φ_1 and φ_2 lead to greater geometric stability.

In the impact diagram based on the angular analysis of the hodograph, under some level of system instability, it is also observed that smaller adjustment angles φ_1 and φ_2 provide greater geometric stability and should lead to improved surface roughness on a dynamically stable machine. In particular, there is a constant local depression about $\varphi_2 = 173^\circ$ ($\gamma = 7^\circ$), which may be the most optimal condition for centerless grinding. There are several local minima in the diagram, which indicate the presence of several acceptable geometric debugging conditions that provide regulated waviness of the detail.

If the machine is dynamically stable from the point of view of vibration, then according to the general diagram of geometric stability it is possible to predict trends of final parameters of waviness on the basis of geometry of adjustment of rigid supports at centerless mortise grinding. The results of this study create the preconditions for controlling the parameters of the waviness of bearing detail, it will stabilize bearing parameters such as noise and vibration.

References

1. Junbo, T., Weining, L., Juneng, A., Xueqian, W.: Fault diagnosis method study in roller bearing based on wavelet transform and stacked auto-encoder. The 27th Chinese Control and Decision Conference (2015 CCDC), pp. 4608–4613. (2015). <https://doi.org/10.1109/CCDC.2015.7162738>
2. Zhang, D., Chen, Y., Guo, F., Karimi, H. R., Dong, H., Xuan, Q.: A New Interpretable Learning Method for Fault Diagnosis of Rolling Bearings. In IEEE Transactions on Instrumentation and Measurement. vol. 70. pp. 1–10. (2021). <https://doi.org/10.1109/TIM.2020.3043873>
3. Adrian, C., Daniela, P.: The study of kinetic energy variation during centerless grinding in the conditions of adaptive control. 2016 International Conference on Applied and Theoretical Electricity (ICATE). pp. 1–6. (2016). <https://doi.org/10.1109/ICATE.2016.7754663>
4. Yakimov, A., Novikov, F., Novikov, G., Serov, B., Yakimov, A.: Theoretical foundations of cutting and grinding materials. Textbook. OGPU, Odessa (1999).
5. Zablotskyi, V., Tkachuk, A., Moroz, S., Prystupa, S., Svirzhevskyi, K.: Influence of Technological Methods of Processing on Wear Resistance of Conjugated Cylindrical Surfaces. In: Tonkonogiy V. et al. (eds) Advanced Manufacturing Processes II. InterPartner 2020. Lecture Notes in Mechanical Engineering. Springer, Cham pp. 477–487 (2021). https://doi.org/10.1007/978-3-030-68014-5_47
6. Fedotkin, I.: Mathematical modeling of technological processes. Textbook. Librocom, Moscov (2016).

7. Kalchenko, V., Yeroshenko, A., Oyko, S.: Mathematical modeling of abrasive grinding working process. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu* 6, pp. 76–82 (2017).
8. Chalyj, V., Moroz, S., Ptachenchuk, V., Zablotskyj, V., Prystupa, S.: Investigation of Waveforms of Roller Bearing's Working Surfaces on Centerless Grinding Operations. In: Ivanov V. et al. (eds) *Advances in Design, Simulation and Manufacturing III. DSMIE 2020. Lecture Notes in Mechanical Engineering, Volume 1*, pp. 349–360. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-50794-7_34
9. Zhang, X. -M., Zhang, Q. -J.: Research on the simulation of centerless grinding process. *Proceedings of the 29th Chinese Control Conference*, pp. 5310–5313. (2010).
10. Chunjian, H., Qiuju, Z., Yubing, X.: Research on the Computer Simulation Technique of Cylindrical Centerless Grinding Process. *2010 Second International Workshop on Education Technology and Computer Science*. pp. 431–433. (2010). <https://doi.org/10.1109/ETCS.2010.551>
11. Albizuri J., Fernandes M. H., Garitaonandia I., Sabalza X., Uribe-Etxeberria R., Hernández J.M. An active system of reduction of vibrations in a centerless grinding machine using piezoelectric actuators // *Int. J. Mach. Tools Manuf.* №47. pp.1607–1614. (2007).
12. Yaroshevich, N., Yaroshevych, O., Lyshuk, V.: Drive Dynamics of Vibratory Machines with Inertia Excitation. In: Balthazar J.M. (eds) *Vibration Engineering and Technology of Machinery. Mechanisms and Machine Science*, Springer, Cham Volume 95, pp. 37–47 (2021) https://doi.org/10.1007/978-3-030-60694-7_2
13. Yang, H., Zhang, L., Li, D., Li, T.: Modeling and analysis of grinding force in surface grinding. *2011 IEEE International Conference on Computer Science and Automation Engineering*, pp. 175–178. (2011). <https://doi.org/10.1109/CSAE.2011.5952448>
14. Jian, C., Wei, D., Yu, Z.: Research on process reliability of grinding based on machining physics. *2014 Prognostics and System Health Management Conference (PHM-2014 Hunan)*. pp. 546–552. (2014) <https://doi.org/10.1109/PHM.2014.6988232>
15. Chetverzhuk, T., Zabolotnyi, O., Sychuk, V., Polinkevych, R., Tkachuk, A.: A method of body parts force displacements calculation of metal-cutting machine tools using CAD and CAE technologies. *Annals of Emerging Technologies in Computing (AETiC)*, pp. 37–47, Vol. 3, No. 4, (2019) <https://doi.org/10.33166/AETiC.2019.04.004>
16. Zan, T., Pang, Z., Wang, M., Gao, X.: Research on Early Fault Diagnosis of Rolling Bearing Based on VMD. *2018 6th International Conference on Mechanical, Automotive and Materials Engineering (CMAME)*. pp. 41–45. (2018). <https://doi.org/10.1109/CMAME.2018.8592450>
17. Karpat, F., Kalay, O.C., Dirik, A.E., Doğan, O., Korcuklu, B., Yüce, C.: Convolutional Neural Networks Based Rolling Bearing Fault Classification Under Variable Operating Conditions. *2021 International Conference on INnovations in Intelligent SysTems and Applications (INISTA)*, pp. 1–6. (2021). <https://doi.org/10.1109/INISTA52262.2021.9548378>