

Mathematical model of a backlash elimination process in the new clamping mechanism of machines

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Abstract. A theoretical model of the operation of the electromechanical mechanism for clamping cylindrical workpieces and tools on the spindle of metalworking machines during backlash elimination is considered. The considered design of the clamping mechanism is driven by an electric motor and has a number of advantages. On the basis of the model, analytical dependencies are proposed that describe the relationship between the movements of the elements of the mechanism during the first stage of the clamping process. The proposed analytical dependencies allow taking into account stages of the functioning of the mechanism when the forces of dissipative characters prevail in its work. The presented dependencies describe the position of the elements of the mechanism depending on the generalized coordinate, which is the angle of rotation of the electric motor rotor. This allows determining the position of the elements of the mechanism depending on time. The obtained dependencies are a precondition for the development of methods for optimizing the design of mechanisms of this type.

Keywords: Clamping Mechanism, Kinematic Parameters, Model.

1 Introduction

As is known, that machining of workpieces on metalworking machines requires relative motion between the workpiece and the tool as well as their force interaction. So, the productivity and quality of the machining directly depend on the characteristics of such shape-forming movements. In order to transfer the shape-forming movements to workpieces or tools, it is necessary to fix them in a spindle assembly of the machine. Clamping mechanisms (CM) are used for this purpose. The characteristics of CM determine the possibility of using the maximum cutting speed and feed rate, and consequently determine productivity and quality of the machining [1]. The magnitude of clamping forces determines the maximum value of the cutting forces that occur during machining. CM of the machines are located on the spindle and are part of the spindle assembly. Hence, requirements for CM are concerning possibilities of creating the required amount of clamping force for a minimum time ensuring its stability in different modes of operation of the spindle assembly. Therefore, the design features of the CM and in particular the geometric and mass parameters of its elements have a significant impact on the characteristics of the whole spindle assembly as well. That

determines the value of resonance (critical) frequency rotation, the possibility of accurate balancing [9] and keeping a balanced state during a given period of operation, etc.

2 Literature Review

There are a number of types of CM with mechanical and hydraulic converters of supplied energy. All the types have a number of disadvantages, including those caused by the presence of movable elements or working medium that can move in the radial direction under the action of centrifugal forces. This leads to a deterioration in the performance of the CM and the spindle assembly as a whole. Modern CM are subject to a number of requirements related to the possibility of their operation in the automatic machining cycle on CNC machines, the efficiency of control and regulation of the clamping force in real-time, reliability, energy efficiency, etc. In order to eliminate the above shortcomings, the design of the electromechanical clamping mechanism with collet chuck has been created (UA Patent No. 95323). The design is simple, with minimum elements, and does not contain ones which move in the radial direction. The use of an electric drive expands the possibilities for creating an effective control system of the new CM. Moreover, since it does not require creating special mechanical (hydraulic) subsystems in the machine for powering the new CM, it provides additional opportunities in the designing of machines. It makes it especially easier to implement the new electromechanical CM into structures of new and existing machines, for example, with the purpose of their modernization.

The characteristics of CM determine the work characteristics of spindle assemblies and consequently productivity and quality of the machining. There are several studies related to the modelling of the processes of spindle assemblies as a whole without separation the dependencies that describe the operation characteristic of the clamping mechanism. There are studies concerning the operation of the clamping chuck [16], spindle bearing carries [7, 15], thermal influence [8] and clamping forces [2, 4]. In researches [6, 10] the expediency of cooling for high productivity of processing is considered. In [13] the research of characteristics of the electric motor operation as a part of the clamping mechanism with the corresponding type of load and the locking (stop-braking) mode of operation has been conducted. Some problems of optimization of a design of spindle units are considered [5, 18]. The influence of the characteristics of the tool [3] and the workpiece [17] on the quality of processing is also considered. There are studies [12, 14] concerning the dynamic characteristics and energy efficiency [11] of the spindle assembly. Identification of interdependences of movement of the elements of the proposed CM is a prerequisite for optimizing its characteristics and consequently for improving the machining.

3 Research Methodology

The operation of the CM is based on the work of an electric motor (see Fig. 1) whose stator is attached to the body of the machine and the rotor is located on the threaded

surface of the spindle. The rotor and spindle form a screw-type transmission, and during the rotation of the rotor its axial movement is used to tighten the collet chuck for clamping the workpiece. The rotation of the rotor (nut) with an angular velocity ω_δ provides its axial motion with a speed V_m transmitted to the tie-rod of the collet.

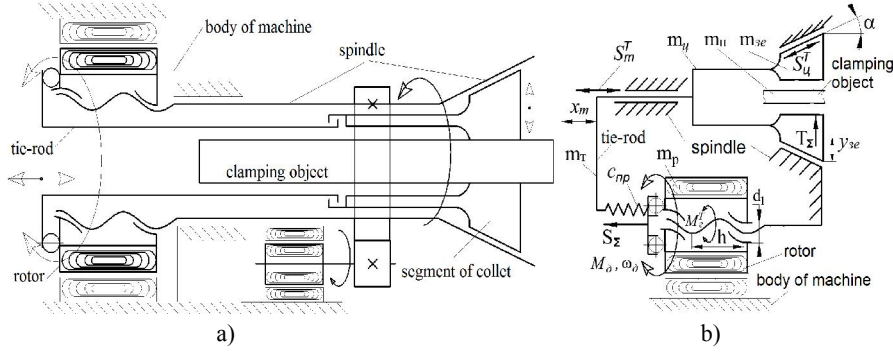


Fig. 1. Constructive scheme of CM (a) and scheme for calculation of parameters (b).

The clamping process can be conditionally divided into two stages according to the type of forces (dissipative or potential) acting in this CM at a certain stage of the clamping process:

- backlash elimination in the CM – only active dissipative forces act;
- creation of mechanical stress (tension) – the action of potential forces prevail owing to significant deformation of an elastic system of the mechanism with the presence of insignificant forces of dissipative character.

The relationships among the kinematic parameters of the proposed CM design can be identified on the basis of the equality between the ratio of the angle ϕ_δ of the engine rotor rotation and the axial movement x_m of the tie-rod connected to the collet.

$$x_m = \frac{\phi_\delta \cdot h}{2\pi} \quad (1)$$

where h – the pitch of the thread. After differentiation of equation (1) regarding time t , we obtain $\frac{\partial x_m}{\partial t} = \frac{\partial \phi_\delta}{\partial t} \cdot \frac{h}{2\pi}$ and

$$V_m = \omega_\delta \cdot \frac{h}{2\pi} = 0,5\omega_\delta \frac{h}{\pi} = \omega_\delta \cdot 0,5d_1 \cdot \text{tg}\Psi \quad (2)$$

where d_1 – the diameter of the thread, Ψ – the angle of rise of the thread. After representing $0,5d_1 \cdot \text{tg}\Psi = i_z$ and putting into (2) we get $V_m = \omega_\delta \cdot i_z$ or

$$V_m = \dot{\phi}_\delta \cdot i_z \quad (3)$$

The movement of the collet in the axial direction with the speed V_m provides radial movement of its clamping segments with the speed V_{3e} . The correlation between V_m and V_{3e} can be found on the basis of the equality between the ratios of the axial displacement x_m of the tie-rod with collet and the radial displacement y_{3e} of the clamping segments $x_m = y_{3e} \cdot ctg\alpha$. From this dependence it follows that

$$y_{3e} = \frac{x_m}{ctg\alpha}, \quad (4)$$

$\frac{\partial x_m}{\partial t} = \frac{\partial y_{3e}}{\partial t} \cdot ctg\alpha$, $V_m = V_{3e} \cdot ctg\alpha$, α - half the angle of a collet cone, $ctg\alpha = i_y$ then

$$V_{3e} = \frac{V_m}{i_y}. \quad (5)$$

After we put (3) in (5).

$$V_{3e} = \frac{\dot{\phi}_o \cdot i_c}{i_y} = \dot{\phi}_o \frac{i_c}{i_y} \quad (6)$$

The frictional torque M_c^T in the screw transmission appears from the tangential component F_t of the friction forces with arm $0,5d_1$. Force F_t depends on the axial load S_Σ acting from the clamping chuck $F_t = S_\Sigma \cdot tg\phi^T$, where ϕ^T - the angle of friction in the threaded connection.

$$M_c^T = 0,5 \cdot d_1 \cdot S_\Sigma \cdot tg\phi^T \quad (7)$$

The friction force S_u^T in the conical coupling of the collet and the spindle with angle 2α is determined from the expression

$$S_u^T = \frac{z \cdot S_\Sigma}{\cos\alpha + (\sin\alpha)/f_u} \quad (8)$$

where z - the number of collet clamping segments; f_u - coefficient of friction in the conical coupling of the collet and the spindle.

The friction force S_m^T in the coupling of the tie-rod and the spindle is determined from the expression

$$S_m^T = g \cdot m_2 \cdot f_m \quad (9)$$

where g - free-fall acceleration; $m_2 = m_m + m_n$ - mass equal to the sum of the masses of the tie-rod and object of clamping; f_m - coefficient of friction in the cou-

pling of the tie-rod with the spindle.

The considered mechanism has one degree of freedom ϕ_o – the angle of rotation of the rotor on which the position of the elements of the CM depends. The angle ϕ_o is taken as a generalized coordinate relative to which the differential equation of motion of the system – the Lagrange's equations of the 2nd kind is composed

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\phi}_o} \right) - \frac{\partial E}{\partial \phi_o} = M_{\Pi} \quad (10)$$

where E – is the kinetic energy of the mechanism; M_{Π} – generalized force, which in this case is equal to the main torque of external forces relative to the axis of the rotor rotation; t – time.

At both stages of working the CM, its kinetic energy E does not depend on the angle ϕ_o of the rotor rotation, so the partial derivative of the kinetic energy E on the angle ϕ_o is

$$\frac{\partial E}{\partial \phi_o} = 0. \quad (11)$$

For the conditionally selected first stage of operation of the CM, when at the stage of backlash elimination there are only non-conservative (non-potential) forces, the Lagrange equation (10) considering (11) takes the form.

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\phi}_o} \right) = M_{\Pi} \quad (12)$$

The kinetic energy E of the CM is equal to the sum of the kinetic energies of its elements and is determined by the expression

$$E = J_o \frac{\dot{\phi}_o^2}{2} + m_1 \frac{V_m^2}{2} + m_{3e} \frac{V_{3e}^2}{2} \quad (13)$$

where $m_1 = m_p + m_m + m_u + m_n$, J_o – the moment of the rotor inertia relative to the axis of rotation; ϕ_o – the rotation angle of the rotor; V_m , V_{3e} – linear velocity of the tie-rod (and other elements with which it has the possibility of direct force interaction), the movement of the clamping segments in the radial direction; m_p , m_m , m_u , m_n – respectively, the mass of the rotor, tie-rod, collet and clamping object (assumed that the clamping object at the time of the clamping moves with the collet); m_{3e} – the mass of the clamping segments of the collet, which move radially during clamping.

After (3) and (6) put in (13) $E = J_\delta \frac{\dot{\phi}_\delta^2}{2} + m_1 \frac{\dot{\phi}_\delta^2}{2} i_z^2 + m_{3e} \frac{\dot{\phi}_\delta^2}{2} \left(\frac{i_z}{i_y} \right)^2$

$$E = \frac{\dot{\phi}_\delta^2}{2} \left[J_\delta + m_1 i_z^2 + m_{3e} \left(\frac{i_z}{i_y} \right)^2 \right] = \frac{\dot{\phi}_\delta^2}{2} J_\Pi. \quad (14)$$

In equation (14), the value in square brackets is the moment of inertia J_Π of the CM reduced to the motor rotor.

$$J_\Pi = \left[J_\delta + m_1 i_z^2 + m_{3e} \left(\frac{i_z}{i_y} \right)^2 \right] \quad (15)$$

Therefore (13) takes the form.

$$E = \frac{\dot{\phi}_\delta^2}{2} J_\Pi \quad (16)$$

The partial derivative of the expression of the kinetic energy E (16) at the generalized velocity $\dot{\phi}_\delta$ has the form $\frac{\partial E}{\partial \dot{\phi}_\delta} = \dot{\phi}_\delta J_\Pi$. Due to the fact that in the previous ex-

pression when changing the time t changes only the angle ϕ_δ , the time derivative of the partial derivative of the previous expression will be equal to

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\phi}_\delta} \right) = \ddot{\phi}_\delta J_\Pi. \quad (17)$$

According to (17), expression (12) can be written in the following form.

$$\ddot{\phi}_\delta J_\Pi = M_\Pi \quad (18)$$

The generalized force M_Π (as the reduced moment) can be determined, taking into account the friction in the CM on the condition of equality of elementary works of forces acting in the mechanism as virtual works of active forces on small displacements

$$M_\Pi \delta \phi_\delta = M_\delta \delta \phi_\delta - M_z^T \delta \phi_\delta - \delta x_m (S_m^T + S_y^T + S_{ny}) - T_\Sigma \frac{1}{3} \delta y_{3e} \quad (19)$$

where M_z^T and S_m^T , S_y^T , S_{ny}^T – the torque of friction forces in the screw transmission and the friction forces in the couplings respectively: the tie-rod with the spindle; the

cone of the collet with the cone of the spindle; a clamping object with the feed collet; T_{Σ} – clamping force; $\delta\phi_o$ and, δx_m , δy_{3e} – elementary angle of rotation in the direction of action M_{Σ}^T and movement in the direction of action, respectively S_m^T , T_{Σ} .

In dependencies (19), to simplify the calculations, the following assumptions are made. Due to the insignificant value of the angle α , the value of the relative displacement of the cone surfaces of the collet and the spindle is equal to the magnitude of the stroke of the tie-rod δx_m . The part of the radial displacement of the clamping segment, which determines the effective work on the clamping of the workpiece (deformations in contacts of adjoining the surfaces of a clamping segment and an object of clamping) with force T_{Σ} is less than the total value of the radial displacement y_{3e} of the clamping segment during the clamping process and depends on conditions of adjoining the surfaces of a clamping segment and an object of clamping. Therefore, in (19) the value of the radial displacement of the clamping segment during the action of force T_{Σ} and increasing mechanical tension in the coupling of a clamping segment and an object of clamping is conditionally assumed to be equal $0.3y_{3e}$. Some possible exaggeration of the value of work from the action of the force T_{Σ} , which resulting from this assumption, is partially offset in the overall balance of elementary works (8) by the presence of other elementary works (energy losses) that inevitably exist during the operation of the real CM.

4 Results

The relationship between the elementary angle $\delta\phi_o$ of rotation of the rotor and the displacements $\delta x_m = \delta\phi_o \frac{h}{2\pi}$ and $\delta y_{3e} = \delta x_m \cdot \text{tg}\alpha$ is represented by analogy with (1) (4), therefore $\delta y_{3e} = \delta\phi_o \frac{h}{2\pi} \cdot \text{tg}\alpha$. The forces included in (19) are presented in (7) (8) (9). The force of pushing the workpiece through the feed collet is determined from the expression $S_{m_i} = P_{m_i}(1 + K\Delta d)$, where P_{m_i} – the pushing force of the workpiece with a diameter equal to the diameter of the hole in the feed collet; $K = A/P_{m_i}$ – coefficient of proportionality; A – change of force of pushing of a workpiece at change of its diameter on each 0.2mm.

After putting (7), (8) and (9) into (19), we get

$$M_{II} = M_o - 0,5d_1 S_{\Sigma} \text{tg}\phi^T - \frac{h}{2\pi} \left(gm_2 f_m + \frac{zS_{\Sigma}}{\cos\alpha + \sin\alpha/f_y} + P_{m_i}(1 + K\Delta d) \right) - T_{\Sigma} \left(\frac{1}{3} \frac{h}{2\pi} \text{tg}\alpha \right) \quad (20)$$

where $S_{\Sigma} = \frac{2M_o}{d_1 \text{tg}\psi}$, $\frac{h}{\pi} = d_1 \text{tg}\psi$.

Dependence (10) describes the system of non-conservative forces of the CM because of the absence of elastic forces at the stage of backlash elimination and it makes it possible to make assumptions that before the appearance of tension the dependence $T_{\Sigma} = T_{\Sigma}^0 = k \cdot y_{3e}$ is linear. There is a slight increase of T_{Σ}^0 with magnification y_{3e} (small angle of the graph of the function), where k – a constant value that reflects the intensity of increasing the clamping force from the radial movement of a clamping segment and can be selected in accordance with the operating conditions of the CM.

$$M_{II} = M_o \left(1 - \frac{tg\phi^T}{tg\psi} - \frac{z}{\cos\alpha + \sin\alpha/f_u} \right) - \frac{h}{2\pi} (gm_2 f_m + P_{mu} (1 + K\Delta d)) - T_{\Sigma}^0 \frac{h}{6\pi} tg\alpha \quad (21)$$

Considering (4) $T_{\Sigma}^0 = \phi_o \frac{k \cdot h}{2\pi} tg\alpha$ and $\frac{h}{\pi} = d_1 tg\psi$ are put into (21) and two first summands that do not contain variable are denoted by symbols θ_1 and β_1 .

$$\theta_1 = M_o \left(1 - \frac{tg\phi^T}{tg\psi} - \frac{z}{\cos\alpha + \sin\alpha/f_u} \right) - \frac{h}{2\pi} (gm_2 f_m + P_{mu} (1 + K\Delta d)) \quad (22)$$

$$\beta_1 = -\frac{k \cdot h^2}{12\pi^2} tg^2\alpha = -\frac{k}{12} d_1^2 tg^2\psi \cdot tg^2\alpha \quad (23)$$

$$M_{II} = \theta_1 + \beta_1 \phi_o \quad (24)$$

Substituting (24) for (18) we obtain a differential equation of the following form.

$$\ddot{\phi}_o J_{II} = \theta_1 + \beta_1 \phi_o \quad (25)$$

Reassignment in (25) $\theta = \frac{\theta_1}{J_{II}}$ and $\beta = \frac{\beta_1}{J_{II}}$.

$$\ddot{\phi}_o = \theta + \beta \phi_o \quad (26)$$

When $\frac{d\phi_o}{dt} = \omega$ than $\frac{d^2\phi_o}{dt^2} = \frac{d\omega}{dt} = \frac{d\omega}{d\phi_o} \cdot \frac{d\phi_o}{dt} = \frac{d\omega}{d\phi_o} \omega$ and consequently

$\frac{d\omega}{d\phi_o} \omega = \theta + \beta \phi_o$; $\int \omega d\omega = \int (\theta + \beta \phi_o) d\phi_o$; $\frac{\omega^2}{2} = \theta \phi_o + \beta \frac{\phi_o^2}{2} + C_1$. The constant of

integration C_1 is defined from initial conditions. At the initial moment of time the speed of the rotor relative to the spindle is equal to zero (but the rotor relative to the stator can rotate with the velocity of the spindle) $\phi_o = 0$, $\omega = 0$, $C_1 = 0$ consequently

$$\frac{d\phi_o}{dt} = \sqrt{\beta \phi_o^2 + 2\theta \phi_o} \quad \text{or} \quad \int \frac{d\phi_o}{\sqrt{\beta \phi_o^2 + 2\theta \phi_o}} = \int dt + C_2 \quad \text{and}$$

$\int \frac{d\phi_o}{\sqrt{\beta\phi_o^2 + 2\theta\phi_o}} = \frac{1}{\sqrt{\beta}} \ln \left[2\beta\phi_o + 2\theta + 2\sqrt{2\theta\beta\phi_o + \beta^2\phi_o^2} \right]$. The solution of the previous

equation can be written as follows $\ln \left[2\beta\phi_o + 2\theta + 2\sqrt{2\theta\beta\phi_o + \beta^2\phi_o^2} \right] = \sqrt{\beta} \cdot t + C_2$.

The constant C_2 can be found from the initial condition when $t = 0$, $\phi_o = 0$. Consequently $\ln 2\theta = C_2$. Substitute in the previous expression

$$\frac{\beta}{\theta}\phi_o + \sqrt{2\frac{\beta}{\theta}\phi_o + \frac{\beta^2}{\theta^2}\phi_o^2} = e^{\sqrt{\beta} \cdot t} - 1.$$

$$\phi_o = \frac{\left(e^{\sqrt{\beta} \cdot t} - 1 \right)^2}{2\frac{\beta}{\theta} e^{\sqrt{\beta} \cdot t}} \quad (27)$$

The value β is characteristic because its positive values determine the monotonic increase of the function (27) and in the case of negative values – local declines and rises (fluctuations) are possible. Equation (27) describes the dependence of the generalized coordinate ϕ_o on time at the stage of elimination of backlashes (under the action of non-conservative forces) in the CM, which makes it possible to determine motion characteristics of the CM elements through dependences describing their position depending on ϕ_o .

5 Conclusions

The obtained mathematical dependencies reflect the patterns of movement of the elements of the clamping mechanism expressed by using a generalized coordinate in the form of the angle of rotation of the motor rotor. The peculiarities of the mechanism operation at the stage of backlash elimination are considered. The received mathematical dependencies reflect the influence on the operation of the mechanism of forces of friction, mass and geometrical characteristics of its elements. The obtained dependencies are a prerequisite for the development of methods for optimizing the parameters of the elements of mechanisms of this type.

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